Theoretical Computer Science 531 (2014) 54–65

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Complexity and approximation for Traveling Salesman Problems with profits

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ARTICLE INFO

Article history:
Received 17 May 2011
Received in revised form 30 January 2014
Accepted 24 February 2014
Communicated by T. Erlebach

Keywords:
Routing problem with profit
Computational complexity
Approximation algorithm
Path
Tree
Cycle

ABSTRACT

In TSP with profits we have to find an optimal tour and a set of customers satisfying a specific constraint. In this paper we analyze three different variants of TSP with profits that are NP-hard in general. We study their computational complexity on special structures of the underlying graph, both in the case with and without service times to the customers. We present polynomial algorithms for the polynomially solvable cases and fully polynomial time approximation schemes (FPTAS) for some NP-hard cases.

1. Introduction

A huge number of papers has studied the well known Traveling Salesman Problem (TSP). In the TSP all customers are to be visited. There exist, however, several practical situations where customers have to be selected among a set of potential customers on the basis of their profits. The goal is to find an appropriate trade-off between the total collected profit and the cost of the tour. A survey by Feillet et al. [13] defines these problems as the *Traveling Salesman Problems with profits*. The objective function may be the difference between the total profit collected and the cost of the tour (Profitable Tour Problem or PTP), the maximization of the total collected profit (Orienteering Problem or OP), the minimization of the cost of the tour (Prize-Collecting TSP or PCTSP). These problems are all NP-hard in general. To further enlighten the computational complexity of these problems, we investigate their complexity on some classes of instances, characterized by

- customers located on a path exiting from the depot;
- customers located on a cycle;

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1 Research supported in part by the Hungarian Scientific Research Fund, OTKA grant T-81493.

http://dx.doi.org/10.1016/j.tcs.2014.02.046
0304-3975/

- customers located on a star rooted at the depot;
- customers located on a tree rooted at the depot.

These classes of instances correspond to special structures of the underlying graph, namely a path, a cycle, a star, a tree.

We study the computational complexity of the problems on these classes of instances, both in the case a service time is associated with each customer and in the case without service times. We present polynomial algorithms for the polynomially solvable cases and fully polynomial time approximation schemes for the NP-hard cases. We also present a reduction to eliminate service times in general networks.

In the rest of the introduction we define the problems we study, present earlier results and summarize the results of this paper.

In Section 2 we prove that the TSP with profits we study are intractable already on some rather restricted classes of instances. These results justify the interest in some even more restricted network topologies. Moreover, at the end of this section, we present a reduction lemma which transforms a problem instance with service times to one without. In Section 3 we prove solvability in linear time of four cases from which we derive the linear time solvability of a number of special cases. Fully polynomial time approximation schemes are presented in Section 4 for OP and PCTSP on tree networks that can be directly applied to star and path networks and easily adapted to cycle networks.

1.1. Problem definitions

We assume that a road network is represented by a connected weighted graph $G = (V, E)$ where the vertices $V = \{i \mid i = 0, \ldots, n\}$ represent road intersections and locations of relevant points like the depot and the customers. We assume throughout that the depot is indexed by 0 and other vertices are indexed from 1 to n. The edges represent physical connections between pairs of vertices (i, j) and their weights $t_{i,j}$ represent the traveling time from vertex i to vertex j. In general, visiting a customer i requires a service time $s_i \geqslant 0$ and offers a profit $p_i \geqslant 0$; nevertheless, a customer vertex in the network can be traversed without visiting it. This means that no service time is spent and no profit is collected. For sake of uniformity we consider road intersection vertices as customers with no profit and zero service time. In this paper we assume all parameters $t_{i,j}$, s_i and p_i to be integers.

The problem can be formulated in different versions. In all cases a subset of customers $S \subseteq \{1, \ldots, n\}$ has to be selected in order to optimize different objective functions and satisfy different constraints involving both the total profit $P_S = \sum_{i \in S} p_i$ and the smallest possible time, denoted by TSP_S, of visiting all the customers in S, which is calculated from the minimum time length of a tour starting from (and returning to) the depot plus the total service time $ST_S = \sum_{i \in S} s_i$.

We recall the different variants of the problem as reported in the literature (see, for instance, Feillet et al. [13]).

Profitable Tour Problem – PTP

$$\max(P_S - TSP_S)$$

Orienteering Problem – OP

$$\max(P_S)$$

such that

$$TSP_S \leqslant T_{\max}$$

where $T_{\max}$ is given.

Prize Collecting TSP – PCTSP

$$\min(TSP_S)$$

such that

$$P_S \geqslant P_{\min}$$

where $P_{\min}$ is given.

It is worth noticing that there is not a general agreement in the literature about problem naming and formulation. Namely, what is called Profitable Tour Problem (PTP) by Feillet et al. [13] is called Prize-Collecting problem by authors like Bienstock et al. [7]. Moreover, PTP is sometimes formulated as a minimization problem: $\min(TSP_S + P_{\bar{S}})$ trying to find a tour that minimizes the cost and the profit missed for not visiting vertices in S. As pointed out in Johnson et al. [18], the two formulations are equivalent as optimization problems (complementing each other), but they are not when approximation algorithms are concerned. From now on, in this paper, we follow the terminology proposed above and refer to the minimization formulation of PTP as $\text{PTP}_{\min}$.