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The diagnosability of triangle-free graphs

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ABSTRACT

The ability of identifying all the faulty devices in a multiprocessor system is known as diagnosability. The local diagnosability concerns the local connective substructure in a network. The PMC model is the test-based diagnosis with a processor performing the diagnosis by testing the neighboring processors via the links between them. In this paper, we discuss the diagnosability and the local diagnosability of a triangle-free network under the PMC model. We also propose the local diagnosis algorithms under the PMC model for some specific structures.

1. Introduction

Sensor networks have gained more and more popularity in computer technology. In a sensor network, there are more than one sensor node or processor. These sensor nodes, processors and the links of a system are usually modeled as a graph topology. The reliability of a system is crucial since even a few malfunctions would disable to operate the service. Whenever devices are found to be faulty, they should be replaced with fault-free ones as soon as possible to guarantee that the system can work properly. Thus the ability of identifying all the faulty devices in a system is very important. This is known as *system diagnosis*. The *diagnosability* is the maximum number of faulty devices that can be identified correctly. A system is *t-diagnosable* if all the faulty devices can be pointed out precisely with the faulty devices being at most t. Many results about the diagnosis and the diagnosability have been proposed in the literature [1–5].

The PMC diagnosis model is proposed by Preparata, Metze, and Chien in [6]. The PMC model is the test-based diagnosis with a processor performing the diagnosis by testing the neighboring processors via the links between them. In [7], Hakimi and Amin proved that a system is t-diagnosable if it is t-connected with at least $2t+1$ nodes under the PMC model. Moreover, they gave a necessary and sufficient condition for verifying if a system is t-diagnosable under the PMC model. Some related works have appeared in the literature [8–11].

In practice, processors in many systems are connected sparsely. Thus some research concerns with the measure that can better reflect fault patterns in real systems. For instance, Das et al. [2] investigated fault diagnosis under local constraints. For a t-diagnosable system, if there exist more than $t+1$ faulty processors, we cannot make a precise diagnosis in this system. However, for the local sense, if the $t+1$ faulty processors are gathered in a particular block of a large

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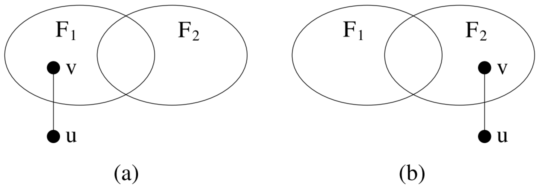

Fig. 1. A distinguishable pair (F_1, F_2).

system, the situation will not affect to the other good parts. Hsu and Tan [12] proposed the concept of local diagnosability to identify the diagnosability of a large system by computing the local diagnosability with respect to each individual processor.

In this paper, we discuss the diagnosability of a graph under the PMC model. We prove that the diagnosability of a triangle-free graph G is the minimum degree of G if G is not isomorphic to a complete bipartite graph. We also concern with the local diagnosability and design a local diagnosis algorithm under the PMC model. The remainder of this paper is organized as follows: Section 2 provides preliminary background for system diagnosis. We prove the diagnosability and the local diagnosability of a graph in Section 3 and Section 4, respectively. We give a local diagnosis algorithm under the PMC model in Section 5. In the final section, we present our conclusion.

2. Preliminaries

The underlying topology of a multiprocessor system is usually modeled as a graph, whose node set and edge set represent the set of all processors and the set of all communication links between processors, respectively. For the graph definitions and notations, we follow [13]. Let $G = (V, E)$ be a *graph* if V is a finite set and E is a subset of $\{\{u, v\} \mid \{u, v\}$ is an unordered pair of $V\}$. We say that V is the *node set* and E is the *edge set* of G. A graph $G = (V_0 \cup V_1, E)$ is *bipartite* if $V_0 \cap V_1 = \emptyset$ and $E \subseteq \{\{u, v\} \mid u \in V_0$ and $v \in V_1\}$. A bipartite graph $G = (V_0 \cup V_1, E)$ is *complete* if for any two nodes, $u \in V_0$ and $v \in V_1$, $\{u, v\}$ is an edge in G. We use $K_{m,n}$ to denote a complete bipartite graph if $|V_0| = m$ and $|V_1| = n$. Two nodes u and v are *adjacent* if $\{u, v\} \in E$; we say u is a *neighbor* of v, and vice versa. We use $N(u)$ to denote the neighborhood set $\{v \mid \{u, v\} \in E(G)\}$. A graph G is *triangle-free* if G contains no cycles of length being three. Note that if u and v are two adjacent nodes in a triangle-free graph, then $N(u) \cap N(v) = \emptyset$. The *degree* of a node v in a graph G, denoted by $\deg_G(v)$, is the number of edges incident to v. We use $\delta(G)$ to denote the minimum degree of G.

Under the PMC diagnosis model, we assume that adjacent processors are capable of performing tests on each other. Let $G = (V, E)$ denote the underlying topology of a multiprocessor system. For any two adjacent nodes $u, v \in V(G)$, the ordered pair (u, v) represents a *test* that processor u is able to diagnose processor v. In this situation, u is a *tester*, and v is a *testee*. The outcome of a test (u, v) is 1 (respectively, 0) if u evaluates v to be faulty (respectively, fault-free). Since the faults considered here are permanent, the outcome of a test is *reliable* if and only if the tester is fault-free. A *faulty set* F is the set of all faulty processors in G. It is noticed that F can be any subset of V. A *test assignment* for system G is a collection of tests and can be modeled as a directed graph $T = (V, L)$, where $(u, v) \in L$ implies that u and v are adjacent in G. The collection of all test results from the test assignment T is called a *syndrome*. Formally, a syndrome of T is a mapping $\sigma : L \to \{0, 1\}$. For any given syndrome σ resulting from a test assignment $T = (V, L)$, a subset of nodes $F \subseteq V$ is said to be *consistent* with σ if for a $(u, v) \in L$ such that $u \in V - F$, then $\sigma(u, v) = 1$ if and only if $v \in F$. This corresponds to the assumption that fault-free testers always give correct test results, whereas faulty testers can lead to unreliable results. Therefore, a given set F of faulty nodes may be consistent with different syndromes. Let $\sigma(F)$ denote the set of all possible syndromes with which the faulty set F can be consistent. Then two distinct faulty sets F_1 and F_2 of V are *distinguishable* if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$; otherwise, F_1 and F_2 are *indistinguishable*. That is, (F_1, F_2) is a *distinguishable pair* of faulty sets if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$. Otherwise, (F_1, F_2) is an *indistinguishable pair*. For any two distinct faulty sets F_1 and F_2 of G with $|F_1| \leqslant t$ and $|F_2| \leqslant t$, a system G is t-diagnosable if and only if (F_1, F_2) is a distinguishable pair. Let F_1 and F_2 be two distinct sets. We use $F_1 \triangle F_2$ to denote the symmetric difference $(F_1 - F_2) \cup (F_2 - F_1)$ between F_1 and F_2. Dahbura and Masson presented a sufficient and necessary characterization of t-diagnosable systems in [9].

Theorem 1. *(See [9].) Let $G = (V, E)$ be a graph. For any two distinct subsets F_1 and F_2 of V, (F_1, F_2) is a distinguishable pair if and only if there exists a node $u \in V - (F_1 \cup F_2)$ and a node $v \in F_1 \triangle F_2$ such that $\{u, v\} \in E$. See Fig. 1 for an illustration.*

Theorem 2. *(See [9].) A graph $G = (V, E)$ is t-diagnosable if and only if, for each distinct pair of sets $F_1, F_2 \subset V$ with $|F_1| \leqslant t$ and $|F_2| \leqslant t$, F_1 and F_2 are distinguishable.*

Chang et al. [8] presented some conditions for a t-regular triangle-free graph to be t-diagnosable as the following theorem.