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Parameterized complexity of Max-lifetime Target Coverage in wireless sensor networks ☆

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ABSTRACT

Max-lifetime Target Coverage can be viewed as a family of problems where the task is to partition the sensors into groups and assign their time-slots such that the coverage lifetime is maximized while satisfying some coverage requirement. Unfortunately, these problems are NP-hard. To gain insight into the source of the complexity, we initiate a systematic parameterized complexity study of two types of Max-lifetime Target Coverage: Max–min Target Coverage and Max-individual Target Coverage. We first prove that both problems remain NP-hard even in the special cases where each target is covered by at most two sensors or each sensor can cover at most two targets. By contrast, restricting the number of targets reduces the complexity of the considered problems. In other words, they are both fixed parameter tractable (FPT) with respect to the parameter "number of targets". Moreover, we extend our studies to the structural parameter "number k of sensors covering at least two targets". Positively, both problems are in FPT with respect to k. Finally, we show that Max–min Target Coverage is in FPT with respect to the combined parameters "number of groups" and "number of targets covered by each group".

1. Introduction

Energy efficiency is a critical issue in wireless sensor networks since sensors are battery powered. Therefore, reducing power consumption and prolonging network lifetime are the primary challenges in the design of wireless sensor networks. In this paper, we address the target coverage problems with the objective of maximizing network lifetime. In recent years, Max-lifetime Target Coverage problems have been studied extensively from the view points of approximation, heuristic, and randomization, etc. In this work, we study new algorithmic approaches based on parameterized complexity analysis for the Max-lifetime Target Coverage problems. We start with introducing the considered problems.

In a randomized deployed sensor network, to monitor a set T of targets with known locations, a large number of sensors S equipped with limited energy supply are dispersed randomly in close proximity to the set of targets. Assume that all the sensors have the same energy supply and can be active for a unit time of 1. If a target $t \in T$ is within the sensing range of a sensor $s \in S$, then we say that t is *covered* (monitored) by s. To increase the likelihood of coverage, the number of deployed sensors is usually higher than optimum required. Hence, a natural question is how to exploit the redundancy in the sensor network to prolong the network lifetime while guaranteeing the *coverage requirement*. One of the most prominent

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Table 1
Parameterized complexity of Max-lifetime Target Coverage.

	Max–min coverage	Max-individual coverage
Number of groups k_1	NP-h for $k_1 = 3$	NP-h for $k_1 = 3$
Number of sensors covering at least two targets k	FPT	FPT
Number of targets k'	FPT	FPT
k_2 or k_3	open	NP-h for $k_3 = 3$
$(k_1, k_2$ or $k_3)$	FPT	NP-h for $k_1 = k_3 = 3$

approaches is to organize the sensors into a number of groups that are activated successively. At any moment in time, only one such group is active for monitoring targets and consumes energy, while other groups are in sleep mode. Then the network lifetime is the total time span of these sensor groups' runtime. Assume that all sensors have the same energy consumption rate in the active state and sensors do not consume energy in sleep state.

Max-lifetime Target Coverage
Input: A set of sensors S, a set of targets T, coverage requirement R, non-negative integer k_1;
Task: Find a family of groups $S_1, \dots, S_{k_1}$ with $S_i \subseteq S$ and associate each group S_i with time t_i $(0 < t_i \leqslant 1)$ such that $\sum_{i=1}^{k_1} t_i$ is maximized under the constraints that the coverage requirement R is satisfied, and every sensor $s \in S$ appears in $S_1, \dots, S_{k_1}$ with a total time at most 1.

Depending on the coverage requirement R, the Max-lifetime Target Coverage problems come in many flavors. If for any two groups S_i, S_j with $i \neq j$, $S_i \cap S_j = \emptyset$, then it is called *disjoint* target coverage, otherwise, it is called *non-disjoint* target coverage. If for every group S_i, S_i must cover all the targets, then it is called *complete* target coverage, otherwise, it is called *partial* target coverage. The partial coverage could usually be one of the following three:

(1) Max–min coverage: Every group S_i must cover at least k_2 targets in T;
(2) Max-individual coverage: For every target $t \in T$, at least k_3 groups of $S_1, \dots, S_{k_1}$ can cover t;
(3) Max-total coverage: The number of total targets covered is at least k_4, i.e., $\sum_{i=1}^{k_1} |T_i| \geqslant k_4$ where T_i denotes the set of targets covered by group S_i.

Note that, for disjoint coverage, to achieve that $\sum_{i=1}^{k_1} t_i$ is maximized, the time-slot assigned to each group must be 1. The coverage lifetime directly corresponds to the number k_1 of groups.

Known results. Cardei et al. [4] showed that disjoint complete coverage is NP-hard, and further proved a lower bound of 2 on approximation ratio. For disjoint max-total coverage, Abrams et al. [1] presented an approximation algorithm with ratio $1 - \frac{1}{e}$. They also proved that it is unlikely to have a polynomial approximation algorithm with ratio better than $\frac{15}{16}$ unless $P = \mathrm{NP}$. For the special case where each target is covered by at most d sensors, Deshpande et al. [12] presented a practical approximation algorithm with ratio $\frac{1}{d} + \frac{2\alpha}{d+2}(1 - \frac{1}{d})$ where α denotes the approximation ratio of the semi-definite programming based algorithm for Max k-Cut. Cheng et al. [8] studied the dual problems of the three kind of disjoint partial coverage problems: the dual problem of max–min coverage requires that, the targets in T uncovered by S_i is at most k_2^d (the other two dual problems can be defined similarly). They proved their NP-hardness and presented heuristic algorithms for them. As to non-disjoint complete coverage, Cheng et al. [7] provided a linear programming based exact algorithm and an approximation algorithm with ratio l (frequency of the most frequently covered target). More results could be found in the literature [24,5,30,28,27].

Our contributions. We initialize a systematic investigation of the hardness of Max-lifetime Target Coverage from the viewpoint of parameterized complexity. We focus on disjoint max–min coverage and disjoint max-individual coverage.

A parameterized problem P is a subset of $\Sigma^* \times \mathbb{N}$ for some finite alphabet Σ, and an instance of a parameterized problem is a pair $(w, k) \in \Sigma^* \times \mathbb{N}$ where k is called the parameter. We say that a parameterized problem is *fixed parameter tractable*, or in FPT, if it can be solved in time $f(k) \cdot \mathrm{poly}(|w|)$ for some computable function f. See [13,21] for more details.

We first reformulate the two considered problems as dominating problems on bipartite graphs and prove the NP-hardness of the dominating problems in the situation where each sensor can cover at most d_1 targets or each target is covered by at most d_2 sensors. We study whether the restriction on d_1 or d_2 would reduce the complexity of the considered problems. Negatively, we show that both problems remain NP-hard even when $d_1 = 2$ or $d_2 = 2$. On the positive side, we show that the two problems can be solved in polynomial time for the case $d_1 = 1$.

We then study the parameterized complexity of the two problems with respect to various parameters. Our results are summarized in Table 1. The parameter "number of groups k_1" does not reduce the complexity of the considered problems because they are NP-hard even when $k_1 = 3$. By contrast, the parameter "number of targets", denoted by k', is more decisive for the hardness of the considered problems because, they are both in FPT with respect to k'. Next, we consider a structural