



Original Research Article

Time delay can enhance spatio-temporal chaos in a prey–predator model

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ABSTRACT

In this paper we explore how the time delay induced Hopf-bifurcation interacts with Turing instability to determine the resulting spatial patterns. For this study, we consider a delayed prey–predator model with Holling type-II functional response and intra-specific competition among the predators. Analytical criteria for the delay induced Hopf-bifurcation and for the delayed spatio-temporal model are provided with numerical example to validate the analytical results. Exhaustive numerical simulation reveals the appearance of three types of stationary patterns, cold spot, labyrinthine, mixture of stripe-spot and two non-stationary patterns, quasi-periodic and spatio-temporal chaotic patterns. The qualitative features of the patterns for the non-delayed and the delayed spatio-temporal model are the same but their occurrence is solely controlled by the temporal parameters, rate of diffusivity and magnitude of the time delay. It is evident that the magnitude of time delay parameter beyond the Hopf-bifurcation threshold mostly produces spatio-temporal chaotic patterns.

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1. Introduction

A wide variety of temporal prey–predator models have been proposed and analyzed to understand the role of prey density and grazing patterns of the predators towards the stable and oscillatory coexistence of both the species. Several types of functional responses (prey-dependent as well as prey and predator dependent) are introduced for the prey–predator type interactions. The classical Rosenzweig–MacArthur model (Rosenzweig and MacArthur, 1963) for prey–predator interaction includes logistic growth for the prey, Holling type-II functional response to describe the grazing pattern of the predators and linear death rate for the predator population. This model basically exhibits two types of coexistence scenario, stable and oscillatory coexistence (Kot, 2001). The model is then modified by Bazykin (1974), introducing an intra-specific competition term into the growth equation of the predator population. The modified model exhibits a wide variety of complex dynamics due to the appearance of more than one coexisting equilibrium point and several local as well as global bifurcation scenarios (Bazykin, 1998; McGehee et al., 2008). Introduction of intra-specific competition term into the predator growth equation leads to logistic type growth for the predators

(Turchin, 2003). Competition among the predators, due to limited resources/favorable habitats, generates a negative feedback to their growth which in turn regulates the growth of prey population and enhances the stable coexistence (McGehee et al., 2008).

Introduction of diffusion terms into the growth equations of prey and predator population leads to the spatio-temporal models in order to take care of the random movement of the individuals of a species within their habitat. The spatio-temporal models are capable of producing more realistic dynamics compared to their temporal counterpart as the mobility of the individuals within their habitat is a relatively realistic assumption. For spatio-temporal models, researchers are interested in understanding the basic mechanisms responsible for the generation of stationary and non-stationary patterns including spatio-temporal chaos. Under certain circumstances, the mobility of the individuals enhances the coexistence scenario due to the appearance of wave of invasion, wave of chaos, and spatio-temporal chaotic dynamics (Banerjee and Abbas, 2015; Banerjee and Petrovskii, 2011; Malchow et al., 2008).

Formation of spatio-temporal patterns due to heterogeneous distribution of the interacting populations is an active area of research during last couple of decades. The idea of heterogeneous population distribution over the habitat of the interacting species was initiated by Levin and Segel (1976) which was based upon the seminal work of Turing (1952) on chemical morphogenesis. Initial research in this area was focused to study the self-organized

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spatial patterns through Turing instability, traveling wave, periodic traveling. [Petrovskii and Malchow \(1999\)](#) first investigated the spatio-temporal prey–predator model with Holling type-II functional response to explain the minimal condition required to be satisfied such that the model can generate spatio-temporal pattern. The model they have considered does not satisfy the criteria for Turing instability. It was explained that the existence of periodic oscillation in the prey and predator species for the temporal model along with the equal diffusivity of both the species and patchy initial population distribution can lead to spatio-temporal pattern formation. Recent investigations (e.g., [Banerjee and Petrovskii, 2011](#); [Tian and Zhang, 2013](#)) ensure that the Hopf-bifurcation in the temporal model is an essential condition for the existence of irregular population distribution over space and this distribution never reaches any stationary distribution.

Recently researchers are interested to study the spatio-temporal pattern formation in the presence of time delay by working on delay-diffusion models. The delay differential equation models are constructed based upon the realistic fact that past history and present population density jointly can influence the population dynamics. This is due to the fact that in most of the interacting populations, change in population density in one species does not respond instantaneously to the growth of the other species rather it is mediated by some time lag. For example, a significant decay in prey population does not deprive the growth of its predator instantaneously. Difference in the strength of competition between the juvenile and adult individuals of the same species, ability of adult individuals to produce new offsprings, time period required for the gestation or incubation, etc. are the examples of natural phenomena responsible for the consideration of delay differential equation models for interacting populations. In general, the introduction of time delay into the interacting population often leads to destabilization but it can capture the oscillatory dynamics of the population density under constant environmental condition. Several types of delay differential equation models are proposed and analyzed for the prey–predator type interactions ([Beretta and Kuang, 1996, 1998](#); [Cao and Freedman, 1996](#); [Cushing, 1977](#); [Freedman and Ruan, 1995](#); [Gopalsamy, 1992](#); [Kuang, 1993](#); [Martin and Ruan, 2001](#); [Ruan, 2001](#); [Sen et al., 2014](#)), here we just refer to some of them. [Hadelor and Ruan \(2007\)](#) studied the joint effect of delay and diffusion on Turing instability for population dynamics and argued that time delay can enhance the spatial and temporal oscillations in population distribution. In couple of literature ([Chen et al., 2013](#); [Lian et al., 2013](#); [Yang, 2007](#)), the authors were interested to derive the Turing instability condition in the presence of time delay and stability of Hopf-bifurcating periodic solution for the spatio-temporal models. The change in resulting spatio-temporal patterns in the presence of time delay is reported in some recent literature ([Hu and Li, 2010](#); [Tian and Zhang, 2013](#); [Zuo, 2013](#); [Zuo and Wei, 2011](#)). In our recent work ([Banerjee and Zhang, 2014](#)), it is established that time delay promotes the spatial patterns through spatio-temporal Hopf-bifurcation.

Spatio-temporal chaotic pattern is able to explain the continuous change of population concentration from one location to other and population distribution over space generates irregular patches. The spatio-temporal chaotic patterns generated by two-species prey–predator type interacting populations can be classified into two groups, namely, interacting spiral patterns and wave of chaos ([Banerjee and Banerjee, 2012](#); [Banerjee and Petrovskii, 2011](#); [Malchow et al., 2008](#)). Interestingly the interacting spiral patterns are not always chaotic rather one can have it as periodic pattern also. Generation of interacting spiral pattern depends upon the intensities of temporal interaction, ratio of diffusivity and the initial population distribution. The spatio-temporal chaotic

patterns are mainly investigated for reaction-diffusion models with self-diffusion.

The goal of this paper is to understand how the instability generated by time delay, involved with the reaction kinetics, influences spatio-temporal chaos by working on a delay diffusion model of prey–predator interaction with Holling type-II functional response and intra-specific competition among the predator population. The main objective of the present paper is to describe two interesting scenarios for the temporal model, corresponding to the spatio-temporal model, where we can find bistable scenario for an intermediate range of parameter values chosen as bifurcation parameter. Based upon these two bifurcation scenarios we are interested in understanding the role of time delay on the spatio-temporal pattern formation. The paper is organized as follows: in Section 2 we describe two bifurcation scenarios for the temporal model with the intensity of intra-specific competition among the predators and the carrying capacity for prey as bifurcation parameters. The Hopf-bifurcation thresholds of the delay for the temporal and spatio-temporal models are provided in Sections 3 and 4 respectively with supportive numerical examples. Section 5 includes gallery of spatio-temporal patterns observed for the model (1) with $\tau = 0$ and $\tau \neq 0$. Significant outcomes of the present research are presented at the concluding section.

2. The model

In this section we first present the delayed spatio-temporal model (1) and then the two bifurcation scenarios for the associated temporal model (2), based on which we will later build to explore the influence of time delay on the formation of spatial patterns.

The delayed spatio-temporal model is described by the following two coupled delay-diffusion equations,

$$\frac{\partial u(t, x, y)}{\partial t} = \rho u(t, x, y) \left(1 - \frac{u(t-\tau, x, y)}{\kappa} \right) - \frac{\beta u(t, x, y)v(t, x, y)}{1 + \alpha u(t, x, y)} + \nabla^2 u(t, x, y), \quad (1a)$$

$$\frac{\partial v(t, x, y)}{\partial t} = \frac{\beta u(t, x, y)v(t, x, y)}{1 + \alpha u(t, x, y)} - v(t, x, y) - \delta v^2(t, x, y) + d \nabla^2 v(t, x, y), \quad (1b)$$

and subjected to the positive initial conditions,

$$u(\theta, x, y) = \psi_1(\theta, x, y) > 0, \quad v(\theta, x, y) = \psi_2(\theta, x, y) > 0$$

for $(\theta, x, y) \in [-\tau, 0] \times \Omega$ and no-flux boundary conditions

$$\frac{\partial u}{\partial n} = 0 = \frac{\partial v}{\partial n}, \quad t \geq 0, (x, y) \in \partial\Omega.$$

Here Ω is a bounded square domain of \mathbb{R}^2 , $\partial\Omega$ is its boundary and 'n' is the outward drawn normal to the boundary. Here $\rho, \kappa, \alpha, \beta$ and δ are all dimensionless positive parameters. κ is the carrying capacity for the prey population and δ represents intra-specific competition coefficients. β and α are, respectively, the attack rate and handling time of predator. 'd' is the ratio of diffusivity for the predator over the prey population (interested readers can see [Banerjee and Banerjee, 2012](#); [McGehee et al., 2008](#) for the details of non-dimensional scheme). ' τ ' is the time delay parameter representing the time lag required for the maturity of prey ([Ruan, 2009](#)). For notational simplicity we will not write the arguments of the functions u and v in rest of this paper.

Several ecological phenomena are responsible for the consideration of delay differential models where the rates of growth,

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