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When less is more: Partial control to avoid extinction of predators in an ecological model

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ABSTRACT

Extinction of a species is one of the most dramatic processes in ecology. Here we use an extended version of the McCann–Yodzis three-species food chain model proposed by Duarte et al. (2009), where a cooperative hunting term was added to the original McCann–Yodzis model and where the three species coexist: resources, consumers and predators. We consider a situation for which a chaotic transient is present in the dynamics implying the predators extinction. Taking into account that the system is affected by external disturbances, we implement a new control method, the partial control method, with the goal of avoiding the extinction with a control applied smaller than the external disturbances of the system. We have also shown that the partial control method implies smaller controls.

1. Introduction

Models of predator–prey systems constitute an important research field in ecology. Different dynamical behaviors, like periodic orbits or strange attractors are frequently present in these models, suggesting the complexity that the interaction between species may reach. From an ecological point of view, one of the most dramatic events occurs when the populations are driven by the dynamics towards an undesirable state. Overpopulation or species extinction are typical situations that may require expensive efforts in the attempt to control the process. In this sense, a good understanding of the underlying causes constitutes a necessary step previous to design a suitable control strategy.

In this work, we consider a particular dynamical behavior called transient chaos. This phenomenon appears in many systems such as a thermal pulse combustor (In et al., 1997), a periodically driven CO_2 laser (Dangoisse et al., 1986), or a voltage collapse (Dhamala and Lai, 1999). The main cause generating this transient behavior from a topological point of view lies at the presence of a chaotic saddle in the phase space. This topological object arises when a chaotic attractor collides with its own basin boundary producing a transient chaotic behavior of trajectories before eventually escaping towards an external attractor (see Sabuco et al., 2010; Lai and Tél, 2011). Likewise, a wide variety of ecological models have been proposed (Hastings and Higgins, 1994; McCann and Yodzis, 1994; Sinha and Parthasarathy, 1996; Gyllenberg et al., 1996; Vandermeer and Yodzis, 1999; Schreiber, 2001) explaining successfully the process of species extinction, as the consequence of the existence of a boundary crisis in phase space. This significant result suggests the importance to deepen in the knowledge of transient chaotic behavior in the context of ecological complexity.

With the purpose of studying the transient chaos in an ecological model, we have chosen a three-species food chain model proposed by Duarte et al. (2009). This model is based on the McCann–Yodzis model (McCann and Yodzis, 1995), where three species coexist: resources, consumers and predators. The interest of this model relies on the simple and plausible explanation of the problem of species extinction, without the necessity to consider temporal or spatial variations and external factors. In addition, the parameters of this model are ecologically meaningful because they were derived from bioenergetics (McCann and Yodzis, 1995).

Following this model, Duarte et al. (2009) have proposed an extended model with the possibility of predators to cooperate to hunt. This behavior has been found in several different situations such as populations involving mammals (Stander, 1991; Mills, 1978), fishes (Cook and Streams, 1984; Major, 1978), insects (Nakasuji and Dyck, 1984) and spiders (Rypstra, 1985; Ward and Enders, 1985), see also Dugatkin (1997). So far, this behavior has typically been modeled as a cooperation strategy in the context of game theory (Packer and Ruttan, 1988), while the theoretical approaches in the context of nonlinear dynamics are weak. With this motivation, Duarte et al. (2009) have added a simple term in the original McCann–Yodzis model, which involves that some

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individuals cooperate during prey's hunting. This term introduces a small Allee effect in the system, that can be adjusted depending on the different degrees of cooperation, recovering the original McCann–Yodzis model when no cooperation exists.

The dynamics of this extended model presents two different behaviors depending on the values of two parameters. In one case, all species coexist in a chaotic attractor, while in the other, transient chaos appears, involving the extinction of the predators population. Duarte et al. (2009) explore how these two states were related with the degree of cooperation, and they found that the extinction of predators is stimulated by his own cooperation strategy, that is, the increase of cooperation drives the predators to the extinction. This relevant result is consistent with recent studies which suggest the importance of intraspecific competition between predators in the stabilization of the dynamics of the three-species food chain models (Deng, 2006).

In the case that the system falls in the extinction state, the question that naturally arises is the possibility to avoid it, sustaining the dynamics in the transient behavior. Following this idea, Duarte et al. (2009) applied the control method described by Dhamala and Lai (1999). They showed that, in absence of disturbances, the transient chaos can be sustained avoiding the predators extinction. However, all real systems are affected by certain external disturbances, producing large deviations in a nonlinear deterministic system (Weis and Knobloch, 1990). In fact, many control methods that are effective without disturbances, can fail when the disturbances are present.

In recent years, a novel control method called *partial control* has appeared in the literature (see Zambrano et al., 2008; Zambrano and Sanjuán, 2009; Sabuco et al., 2012a,b). This control method is applied in situations where transient chaos is present and the system is subjected to external disturbances. The main result of this paper is the successful control of trajectories in the ecological model introduced in Duarte et al. (2009). Furthermore, we show that the amount of control needed is even smaller than in other control strategies.

The structure of the paper is as follows. Section 2 is devoted to the description of the ecological model. The main ideas of the partial control method and its application to the model is described in Section 3. And in Section 4 the safe sets are computed. A comparative analysis of the partial control method with the one used in Duarte et al. (2009), is given in Section 5, where we show that the amount of control needed by using the partial control method for a given amount of noise might be much smaller than in other control methods. Finally, some conclusions are drawn in Section 6.

2. Description of the ecological model

We use a three species food chain model proposed by Duarte et al. (2009). This model is an extension of the McCann–Yodzis model, which describes the dynamics of the population density of a resource species R, a consumer C and a predator P. In addition, Duarte et al. (2009) propose the introduction of a nonlinear term with the aim to model the possibility of the predators to cooperate to hunt. The resulting model is given by the following set of nonlinear differential equations:

$$\begin{aligned}\frac{dR}{dt} &= R\left(1-\frac{R}{K}\right)-\frac{x_c y_c CR}{R+R_0}\\ \frac{dC}{dt} &= x_c C\left(\frac{y_c R}{R+R_0}-1\right)-\psi(P)\frac{y_p C}{C+C_0}\\ \frac{dP}{dt} &= \psi(P)\frac{y_p C}{C+C_0}-x_p P.\end{aligned} \quad (1)$$

The biological assumptions of this model are: (i) continuous growth and overlapping generations are allowed for each species. (ii) The resource population grows logistically. (iii) Consumers and predators dies off exponentially without food. (iv) The feeding rate of consumers and predators saturates at high food levels. Its is important to point out that we are assuming populations big enough to dismiss lattice effects (Henson et al., 2001) which in the case of small populations could change dramatically the dynamics and therefore it should be considered in any control method.

Following McCann and Yodzis (1995), we fix the parameters as: $x_c = 0.4$, $y_c = 2.009$, $x_p = x_i = 0.08$, $y_p = 2.876$, $R_0 = 0.16129$ and $C_0 = 0.5$. The term $\psi(P) = x_p(1-\sigma)P + x_i\sigma P^2$ in the equations represents the reproduction kinetics of predators. In this term Duarte et al. (2009) included the parameter $\sigma \in [0, 1]$, which reflects the fraction of predators that cooperate to hunt. Note that the McCann–Yodzis model is a particular case of this model when $\sigma = 0$.

The interest of this model lies in the fact that the dynamics presents transient chaos depending on the values of the carrying capacity K and the degree of cooperation σ. Analyzing the nonlinear dynamics of the system, it is possible to find the different pair of values (K, σ) for which the boundary crisis takes place. For instance, fixing $K = 0.99$, the boundary crisis appears at $\sigma_c = 0.04166$. This critical value separates the two different dynamical regions. Before the crisis, for $\sigma < \sigma_c$, two attractors coexist in phase space: one chaotic attractor where all the species coexist, and one limit cycle where no predators exist (see Fig. 1a). However, after the crisis (Fig. 1b), the only asymptotic attractor is the limit cycle where no predators exist. Such a crisis becomes the limit cycle in the global attractor. Therefore a trajectory close to the chaotic saddle follows the typical time series represented in Fig. 1c, where initially the predators population has a chaotic transient and then it collapses to zero becoming extinct.

Under these conditions, it is clear that in the absence of an external action, the population of predators is doomed to extinction. In this sense, a question reaches naturally: is it possible to avoid the extinction? Obviously, the answer depends on the realistic possibility to perturb the system to sustain the transient chaos behavior. One approach might be to decrease the resource carrying capacity K or the degree of cooperation between predators σ. However, from an ecological point of view, to change these parameters is not always accessible or takes up a large amount of time acting over the system. Nevertheless, it is possible to modify the dynamics, acting directly on a given dynamical variable. In this sense, we can find in the literature a wide number of control methods, that in different ways, deal with the same problem. We find methods like the "target oriented control" or "proportional feedback" that are able to stabilize the unstable or even the chaotic dynamics around an asymptotic stable equilibrium and have been successfully applied to ecological models (Carmona and Franco, 2011); Dattani et al., 2011; Franco and Peran, 2013). Other strategies, like the "adaptive limiter control", reduce the fluctuations of the populations, with the aim of bounding the dynamics within a certain range and therefore avoiding undesirable or dangerous escapes (Franco and Hilker, 2013). In particular this method preserves the chaotic dynamics, a feature that we consider highly desirable in order to maintain the natural evolution of the ecological system. Finally, we have the method described in Dhamala and Lai (1999) that has also been applied in Duarte et al. (2009), attempting to avoid the extinctions identifying a escape region and not allowing the system to enter inside. Surprisingly, in the literature it is very difficult to find control methods applied to systems with noise, which we consider that is something rather common in all real systems, where the noise will enclose all the uncertainty about the dynamics of the system, like modeling mismatches or external disturbances. In addition, we consider that it would be very helpful to have a method being able to fix an upper control bound that ensures the control of the system regardless of