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Dynamics of a delayed stage-structured model with impulsive harvesting and diffusion

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ABSTRACT

Based on the predator–prey system with delayed stage-structured for preys and impulsive harvesting and impulsive diffusion for predator, an impulsive delayed differential equation to model the process of periodic harvesting and impulsive diffusion at different fixed moments is proposed and investigated. In this model, patches are created by two different prey populations and each prey population is confined to a particular patch while the predator population can impulsively diffuse between two patches. By using comparison theorem of impulsive delayed differential equation and some analysis techniques, sufficient conditions ensuring the existence of preys-extinction periodic solution and the permanence of the system are established. Our analysis reveals that low birth rates of immature preys, high death rates of immature and mature preys, long maturation time of immature preys to mature preys and large preys' captured rates are the sufficient condition for the preys-extinction. On the contrary, if we largen the birth rates of immature preys, or decrease the death rates of immature and mature preys, or shorten the maturation time of immature preys or decrease the preys' captured rates, then the system can become permanent under proper predational strategies. These also show that it is feasible to keep the sustainable development of the ecosystem by controlling the critical ecological parameters. Numerical simulations with hypothetical set of parameter values are carried out to consolidate the analytic findings.

1. Introduction

Population dispersal in patchy environment is one of the most prevalent subjects in ecology and mathematical ecology. Within each patch, individuals of each species are supposed to be identical and can migrate to other patches. In most previous papers, population dynamics with the effects of heterogeneity modeled by the diffusion process is focused on the dynamical system modeled by ordinary differential equations and delay differential equations (Cui et al., 2004; Xu and Ma, 2008; Cui and Chen, 2001; Zhou et al., 2008; Ding and Han, 2008; Chen et al., 2003; Song and Chen, 1998; Xu et al., 2004). But in reality, dispersal behavior is very intricate and is always perturbed by environmental change and human activities, etc. In fact, it is often the case that diffusion occurs during short-time slots within seasons or within the lifetimes of animals. In order to be in much better agreement with the real ecological process, this short-time scale dispersal is more suitable assumed to be in the form of regular pulses. Taking birds as an example, when winter comes, they will diffuse between patches in search for a better environment, but they do not migrate in other season. Thus impulsive diffusion provides a more natural description for this behavior. With the developments and applications of impulsive differential equations, theories of impulsive differential equations have been introduced into population dynamics, and some important studies about impulsive diffusion have been done (Shao, 2010; Jiao et al., 2011, 2011, 2011; Jiao and Cai, 2009a; Hui and Chen, 2005; Wan et al., 2012; Wang et al., 2007; Dong et al., 2007; Zhang et al., 2013; Zhao et al., 2011). In particularly, a single population was considered (Hui and Chen, 2005; Wan et al., 2012; Wang et al., 2007; Dong et al., 2007; Zhang et al., 2013; Zhao et al., 2011). For example, Zhao et al. (2011) studied the following single species model with impulsive diffusion and pulsed harvesting at the different fixed time

$$\begin{cases} \left.\begin{aligned} \frac{dx_1(t)}{dt} &= r_1x_1(t) - a_{11}x_1^2(t) \\ \frac{dx_2(t)}{dt} &= -r_2x_2(t) \end{aligned}\right\} & t \neq nT, \quad t \neq (n+l-1)T \\ \left.\begin{aligned} \Delta x_1(t) &= -Ex_1(t) \\ \Delta x_2(t) &= 0 \end{aligned}\right\} & t = (n+l-1)T \\ \left.\begin{aligned} \Delta x_1(t) &= D(x_2(t) - x_1(t)) \\ \Delta x_2(t) &= D(x_1(t) - x_2(t)) \end{aligned}\right\} & t = nT \end{cases} \tag{1.1}$$

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In this model, suppose that the system is composed of two patches connected by diffusion. $x_i(t)$ presents the biomass of the population in patch $i(i = 1, 2)$. r_1 denotes the intrinsic growth rate in the first patch and r_2 is the death rate in the second patch. r_1/a_{11} is the environment carrying capacity. $0 < E < 1$ is the harvesting effort of the population in the first patch at $t = (n + l - 1)T(0 < l < 1)$. $0 < D < 1$ is the diffusion coefficient. If $x_i > x_j(i \neq j,\ i,\ j = 1,\ 2)$, the population in patch i diffuses into patch j at a rate D which is proportional to $x_i - x_j$. T is the impulsive diffusion period. $\Delta x_i(nT^+) = x_i(nT^+) - x_i(nT)(i = 1,\ 2)$. $x_i(nT^+)(i = 1,\ 2)$ represents the density of the subpopulation in patch i after the nth diffusion pulse at $t = nT$, while $x_i(nT)(i = 1,\ 2)$ represents the density of the subpopulation in patch i immediately before the nth diffusion pulse at $t = nT$. By using the stroboscopic map, they obtained the existence and globally asymptotical stability of both the trivial solution and the positive periodic solution, and the complete expression for the periodic solution.

On the other hand, from birth to death, many species usually go through two life stages, immature and mature. So it is practical to introduce stage-structured into prey–predator models (Aiello and Freedman, 1990; Song and Chen, 2002; Meng et al., 2008; Song and Xiang, 2006; Chen and You, 2008; Shao and Dai, 2010; Huang et al., 2012; Song et al., 2009; Liu et al., 2008; Wang et al., 2009). In additional, as literatures (Li and Kuang, 2001) pointed out that the delay differential equation shows much more complicated dynamics than ordinary differential equation since time delay could cause a stable equilibrium to become unstable and cause the population to fluctuate. Therefore it is reasonable to introduce delayed stage structure into prey–predator models (Meng et al., 2008; Song and Xiang, 2006; Chen and You, 2008; Shao and Dai, 2010; Huang et al., 2012; Song et al., 2009; Liu et al., 2008; Wang et al., 2009). Newly, population dynamical system involving delayed stage structure and impulsive diffusion have been discussed by some authors, see Jiao (2010), Shao and Li (2013), Jiao et al. (2009b), Dhar and Jatav (2013), Jiao et al. (2010), and references cited therein.

Furthermore, in real nature, some lower-order preys can establish their own territory and does not interact with other preys, whereas the predator can diffuse between the territories at a fixed moment. Therefore in this paper, we consider a three-species (two-prey and a predator) ecological model with impulsive harvesting and impulsive diffusion at different fixed moments. To formulate the mathematical model, we make the following assumptions:

A1 The patches are created by two prey populations, and each prey population is confined to a particular patch while the predator population can diffuse between two patches. Predator has two different impulsive time. In the first impulsive time, we only harvest the predator in the first patch. And in the second impulsive time, the predator population will migrate from one patch to other patch.

A2 The prey populations: each prey population have two life stage, namely immature and mature stages. The birth rates into the immature population are proportional to the existing mature population with a proportionality α_1 in patch 1 and α_2 in patch 2, respectively; and the death rates of the immature population are proportional to the existing immature population with a proportionality ω_1 in patch 1 and γ_1 in patch 2, respectively; the death rates of the mature population are proportional to the existing mature population with a proportionality ω_2 in patch 1 and γ_2 in patch 2, respectively; the intra-specific competition rates of the mature prey populations are proportional to square of the population with a proportionality d_1 in patch 1 and d_2 in patch 2, respectively.

A3 The predator population: in the absence of the prey populations, the predator population in the first habitat grows according to the logistic curve with the intrinsic birth rate b_1 and density dependence rate a_1. Whereas the predator subpopulation in the second patch will die with the death rate b_2. The predator populations only feed on the mature preys following Holling type-I functional response with different capturing rates c_1 in patch 1 and c_2 in patch 2, respectively. The conversion factor for predator population due to consumption of prey is $k_i(i = 1, 2)$ in the ith $(i = 1, 2)$ patch.

In the natural world, these assumptions are reasonable for many species whose immature prey population conceal in the cave and are raised by their parents; the rate of predator attacking at immature prey can be ignored. Considering the above basic assumptions, we can derive the following differential equations:

$$\left\{\begin{array}{ll}
\left.\begin{array}{l}
\dfrac{dx_1(t)}{dt} = \alpha_1 x_2(t) - \alpha_1 e^{-\omega_1\tau_1} x_2(t-\tau_1) - \omega_1 x_1(t)\\
\dfrac{dx_2(t)}{dt} = \alpha_1 e^{-\omega_1\tau_1} x_2(t-\tau_1) - \omega_2 x_2(t) - d_1 x_2^2(t) - c_1 x_2(t) z_1(t)\\
\dfrac{dy_1(t)}{dt} = \alpha_2 y_2(t) - \alpha_2 e^{-\gamma_1\tau_2} y_2(t-\tau_2) - \gamma_1 y_1(t)\\
\dfrac{dy_2(t)}{dt} = \alpha_2 e^{-\gamma_1\tau_2} y_2(t-\tau_2) - \gamma_2 y_2(t) - d_2 y_2^2(t) - c_2 y_2(t) z_2(t)\\
\dfrac{dz_1(t)}{dt} = z_1(t)[b_1 - a_1 z_1(t)] + k_1 c_1 x_2(t) z_1(t)\\
\dfrac{dz_2(t)}{dt} = -b_2 z_2(t) + k_2 c_2 y_2(t) z_2(t)
\end{array}\right\} & t \neq nT, \quad t \neq (n+l-1)T\\
\left.\begin{array}{l}
\Delta x_1(t) = 0\\
\Delta x_2(t) = 0\\
\Delta y_1(t) = 0\\
\Delta y_2(t) = 0\\
\Delta z_1(t) = -p z_1(t)\\
\Delta z_2(t) = 0
\end{array}\right\} & t = (n+l-1)T\\
\left.\begin{array}{l}
\Delta x_1(t) = 0\\
\Delta x_2(t) = 0\\
\Delta y_1(t) = 0\\
\Delta y_2(t) = 0\\
\Delta z_1(t) = D(z_2(t) - z_1(t))\\
\Delta z_2(t) = D(z_1(t) - z_2(t))
\end{array}\right\} & t = nT
\end{array}\right. \tag{1.2}$$