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Original Research Article

# Interaction network based early-warning indicators of vegetation transitions

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ABSTRACT

Changes in vegetation patterns in semi-arid regions can precede the abrupt transition to bare soil. Here, complex network techniques are used to develop novel early-warning indicators for these desertification transitions. These indicators are applied to results from a local positive feedback vegetation model and are compared to classical indicators, such as the autocorrelation and variance of biomass time series. A quantitative measure is also introduced to evaluate the quality of the early-warning indicators. Based on this measure, the network-based indicators are superior to the classical ones, being more sensitive to the presence of the transition point.

## 1. Introduction

There are now numerous indications, from observations and models, that transitions between different vegetation states can occur due to the existence of multiple equilibrium states (Scheffer et al., 2001; Rietkerk et al., 2004; Kéfi et al., 2007, 2013). One of these transitions is between vegetated and bare soil states in semi-arid regions and is associated with desertification. Other transitions are, for example, those between savannah and forest states as found in observations of woody cover (Hirota et al., 2011) and above ground biomass (Yin et al., 2014). In the latter case, the equilibrium states appear as different maxima in the probability density function (PDF) of these quantities. Although transient growth effects in the relatively short length of the time series obscure the interpretation of the PDFs, it is plausible that multiple equilibrium states exist (Yin et al., 2014).

The existence of multiple equilibria is supported by models of vegetation dynamics of various levels of complexity. These models can be mainly classified into spatial (PDEs) and non-spatial (ODEs) models, and both types can display multiple equilibria and catastrophic transitions to desertification. Looking at the spatial models, most of them are of reaction-diffusion type, contain positive feedbacks at different scales, represent the dissipation mechanism as diffusion, and use precipitation as stressor. Analysis of three basic spatial models which describe desertification due to decreasing precipitation (Dakos et al., 2011) indicates that the route to desertification occurs through a saddle-node bifurcation which gives rise to hysteresis.

From a land management point of view, these theoretical results suggest that if transitions to desert states occur in a sudden and unexpected way, it would be crucial to develop early-warning signals in order to prevent or at least prepare for such transitions. Previous studies (Scheffer et al., 2009) proposed that such indicators could be developed based on critical slowing down (CSD), that is, the slow recovery of a system to small perturbations which appears to be a generic phenomenon in the vicinity of bifurcation points (Wissel, 1984). However, CSD can only be detected indirectly by specific statistical properties of the dynamics of a system such as an increase in spatial and temporal correlation as well as variance (Scheffer et al., 2009; Guttal and Jayaprakash, 2009; Dakos et al., 2010). These CSD-based statistical properties have been suggested to act as early-warning signals for critical transitions (Scheffer et al., 2009), and they have been experimentally demonstrated to exist in various living systems (Drake and Griffen, 2010; Carpenter et al., 2011; Dai et al., 2012).

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Recently, complex network based measures were proposed as leading indicators of critical transitions in the ocean circulation – also known to be associated with saddle-node bifurcations (Mheen et al., 2013; Viebahn and Dijkstra, 2014). It was shown that an upcoming saddle-node bifurcation can be adequately detected by network measures which are constructed via coarse-graining of the cross-correlation matrix of the spatially extended system.

In this paper, we apply network-based indicators to time series stemming from a vegetation model which exhibits a critical transition to desertification. Both the vegetation model and the network techniques are presented in Section 2. Our aim is to test whether novel network-based measures are able to indicate the upcoming collapse of vegetation to a desert state (Section 3) and to assess their potential as tools for anticipating critical transitions in a wide range of ecological systems (Section 4). The paper is closed with a summary and discussion (Section 5).

## 2. Model and methods

In this section, we first present the spatial vegetation model used to produce biomass time series, and then present the methods of network construction and analysis.

### 2.1. A spatial model of vegetation dynamics with a local positive feedback

The local positive feedback model (LPF) is described by the following set of stochastic differential equations (Shnerb et al., 2003; Guttal and Jayaprakash, 2007; Dakos et al., 2011),

$$\frac{\partial w}{\partial t} = R - \frac{w}{\tau_w} - \Lambda wB + D\nabla^2 w + \sigma_w w_0 \xi^w(t), \tag{1}$$

$$\frac{\partial B}{\partial t} = \rho B\left(\frac{w}{w_0} - \frac{B}{B_c}\right) - \mu\frac{B}{B + B_O} + D\nabla^2 B + \sigma_B B_0 \xi^B(t), \tag{2}$$

where $w$ (in mm) is the soil water amount and $B$ (in g/m$^2$) is the vegetation biomass. The quantity $D$ is the diffusivity and $\tau_w$, $\mu$, $\rho$, $\Lambda$, $w_0$, $B_O$, $B_c$ are additional constants explained in Table 1. Finally, $R$ is the amount of rainfall which is used as the bifurcation parameter of the system. Additive Gaussian white noise, $\xi$, for which

$$\langle \xi(t)\xi(t')\rangle = \delta(t - t'), \tag{3}$$

is prescribed with amplitudes $\sigma_w$ and $\sigma_B$ for soil water and biomass, respectively. A characteristic spatial pattern of biomass as well as time series for different values of $R$ are shown in Fig. 1.

The important mechanism in this model is a positive feedback that causes each patch to have alternative stable states. This is demonstrated in the bifurcation diagram of the LPF model which is sketched in Fig. 2. The deterministic homogeneous solutions of the LPF model and their linear stability can be determined analytically. For all values of $R$, the trivial solution ($B = 0$, $w = \tau_w R$) exists. For the standard parameter values shown Table 1, the trivial solution is linearly stable for $R < 2$ mm/day and unstable for $R > 2$ mm/day (see Fig. 2). At $R = 2$ mm/day, a transcritical bifurcation occurs and two additional branches of steady solutions emerge. Solutions on the lower branch are not considered here because they have $B < 0$, i.e., they are physically non-realistic. Solutions on the upper branch are unstable for values of $R$ down to $R_c = 1.067$ mm/day. At this $R$-value a saddle-node bifurcation occurs which provides a linearly stable upper branch of solutions for $R > 1.067$ mm/day. Finally, a fourth homogeneous solution exists but it has also values of $B < 0$ for every $R$ value and hence is not further considered in this study.

**Table 1**
Parameters of the local positive feedback model (LPF) given by Eq. (2) and same values as in Dakos et al. (2011).

| Parameter | Meaning | Value |
|---|---|---|
| $D$ | Exchange rate (m$^2$/day) | 0.5 |
| $\Lambda$ | Water consumption rate by vegetation (m$^2$/(g day)) | 0.12 |
| $\rho$ | Maximum vegetation growth rate (day$^{-1}$) | 1 |
| $B_c$ | Vegetation carrying capacity (g/m$^2$) | 10 |
| $\mu$ | Maximum grazing rate (g/(day m$^2$)) | 2 |
| $B_O$ | Half-saturation constant of vegetation consumption (g/m$^2$) | 1 |
| $\sigma_w$ | Standard deviation of white noise in water moisture | 0.1 |
| $\sigma_B$ | Standard deviation of white noise in vegetation biomass | 0.25 |
| $w_0$ | Water moisture scale value (mm) | 1 |
| $B_0$ | Biomass density scale value (g/m$^2$) | 1 |
| $\tau_w$ | Water moisture scale time (day) | 1 |

In order to determine inhomogeneous vegetation patterns in the stochastic case, the model equations (2) are numerically solved on a periodic square grid composed of $100 \times 100 = 10^4$ grid cells on a regular lattice with dimension $L = 100$ m. The evaluated model data consists of a set of time series (500 time steps with $\Delta t = 0.01$ days) of statistically equilibrated biomass fields $B$ for different fixed rainfall parameters $R$. Time series related to 10 different values of $R$ with 1.1 mm/day $\leq R \leq$ 1.8 mm/day are analysed. For $R < R_c$ only the desert-like solution, with $B = 0$ over the whole domain, is found.

The temporal and spatial mean values of the biomass distribution of each of these spatially inhomogeneous solutions are plotted as the red dots in Fig. 2. Obviously, the average values of the inhomogeneous solutions are similar to those of the homogeneous solutions, hence, diffusion and noise do not impact on the average state of the system.

### 2.2. Network approach and analysis

A *network* is defined by two sets of objects: the so-called *nodes*, and the set of their mutual connections, namely their *links* (Caldarelli, 2007). In the following, we will associate a network to the simulated field of biomass. The nodes are defined as the $N = 100 \times 100 = 10^4$ grid cells of the discretised LPF model. In order to define the links between the nodes, the zero-lag temporal correlations between the biomass time series at the different nodes are considered. More precisely, two nodes $i$ and $j$ are linked if the temporal cross-correlation $\mathcal{C}(B_i, B_j)$ of the time series $B_i$ and $B_j$ is statistically significant. A sketch of the network formation is depicted in Fig. 3. Note that in this way the number of nodes of the network is kept fixed, and changes in the network structure due to varying $R$ are solely related to the links, that is, to changes in the values of the cross-correlation matrix $\mathcal{C}(B_i, B_j)$.

A compact way to describe a network composed of $N$ nodes is to consider its *adjacency matrix* $\mathbf{A}$, a symmetric $N \times N$ matrix with $A_{ij} = 1$ if node $i$ and node $j$ of the network are linked, and $A_{ij} = 0$ otherwise. Thus, the correlation network of the biomass data is given by the following adjacency matrix,

$$A_{ij} = \mathrm{H}(|\mathcal{C}(B_i, B_j)| - \theta), \tag{4}$$

where H is the Heaviside step function, and $\theta$ is a constant threshold indicating statistical significance of the cross-correlation $\mathcal{C}(B_i, B_j)$.

To determine the value of $\theta$ we build the following test variable for the Student's $t$-test, i.e.,

$$t = \frac{\theta}{\sqrt{1 - \theta^2}}\sqrt{N_{\text{time-steps}}\frac{1 - r}{1 + r}}, \tag{5}$$

with the null hypothesis $\theta = 0$. Here $r = r(R)$ and $N_{\text{time-steps}}$ are the autocorrelation and the length of the time series, respectively. The test variable takes the effective number of degrees of freedom of the time series into account. From this we can compute the value of $\theta$ which ensures statistical significance of correlations larger than