



## Original research article

# Mathematical analysis of a delayed stage-structured predator–prey model with impulsive diffusion between two predators territories

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## ABSTRACT

In most models of population dynamics, diffusion between two patches is assumed to be either continuous or discrete, but in reality, many species diffuse only during a single period, and diffusion often occurs in regular pulses. Further, in forest habitats, the highest-level predator species are restricted to a specific territory, but prey can impulsively move between territories. Therefore, in this paper, we consider a delayed stage-structured predator–prey model with impulsively diffusive prey between two patches; in the model, patches represent the territories of two different predator populations. Here, we analytically obtain the global attractivity condition of predator–extinction periodic solutions for the system by using the concepts of [Hui and Chen \(2005\)](#); a numerical simulation is also included to illustrate this result. Further, we establish permanence conditions for the coexistence of the species using the theory of impulsive delayed differential equations. Finally, we explore the possibilities of the permanence of the system by using the growth rates of immature predators and the impulse period as critical parameters, and we also obtain the parameters' threshold limits using numerical experimentation.

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## 1. Introduction

In the last few years, mathematicians and ecologists have been actively investigating the dispersal of populations, a ubiquitous phenomenon in population dynamics. [Levin \(1976\)](#) showed that both spatial dispersal of populations and population dynamics are much affected by spatial heterogeneity. In real life, dispersal often occurs among patches in ecological environments; because of the ecological effects of human activities and industries, such as the location of manufacturing industries and the pollution of the atmosphere, soil and rivers, reproduction- and population-based territories and other habitats have been broken into patches. Thus, realistic models should include dispersal processes that take into consideration the effects of spatial heterogeneity. Such dispersal processes can be represented by two types of equations: the first type consists of semilinear parabolic equations (i.e., a reaction-diffusion system describing the continuous expansion across the space of the population), and the other type is a discrete diffusion system, where several species are distributed over an interconnected network of multiple patches and the populations migrate among these patches. A large number of mathematical models for

diffusion have been studied by many researchers ([Chen et al., 2003](#); [Cui and Chen, 2001](#); [Cui et al., 2004](#); [Teng and Chen, 2003](#); [Xu and Ma, 2008](#)). However, in real life, we often find that the population diffuses in regular pulses. For example, when winter comes, birds migrate in search of better environments, whereas they do not diffuse during other seasons. Thus, impulsive diffusion provides a more natural behavioural phenomenon for many ecological species.

Impulsive differential equations are used as a mathematical tool to study population models with impulsive diffusion; these models are also used in almost every domain of applied science, not just population dynamics ([Lakmeche and Arino, 2000](#); [Shulgin et al., 1998](#); [Funasaki and Kot, 1993](#); [Liu and Zhang, 1998](#); [Vandermeer et al., 2001](#); [Liu, 1995](#); [Liu and Chen, 2003](#); [Bainov and Simeonov, 1993, 1989](#); [Lakshmikantham et al., 1989](#)). Recently, increased attention has been given to the dynamics of a large number of mathematical models with impulsive diffusion between two patches ([Jiao, 2010a,b](#); [Jiao and Chen, 2008](#); [Jiao et al., 2010, 2009](#); [Wang and Chen, 1997](#); [Shao, 2010](#); [Liu et al., 2010](#); [Wang et al., 2007](#)). The most significant topics in the study of population diffusion models are the coexistence of populations, the local and global stability of all possible equilibria and the existence of periodic solutions. [Beretta and Solimano \(1987\)](#) studied the dynamics of a predator–prey diffusion model in two-patch-involved time delay and obtained a set of sufficient conditions

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for the permanence and stability of positive equilibria. Recently, two different predator–prey systems with either predator- or prey-impulsive diffusion on two patches were studied by Jiao (2010b,a), who obtained sufficient conditions for the global attractivity and permanence of the systems.

In most of the above literature, the researchers assumed that the patches were created due to the environmental gradient and the resource availability. However, sometimes in real life, a higher-order predator is also able to create its own territory and does not interfere with the territory of other predators, whereas the prey population is available everywhere and diffuses between the territories at a fixed moment of time for shelter and resources (Seidensticker, 1996; DuTemple and Stone, 1996). For example, one higher-order predator, the tiger, lives in its own territory, which is larger than a small town, and attacks its prey only if the prey is available in its territory; this behaviour is well-documented in DuTemple and Stone (1996). Moreover, immature predators are concealed in caves and depend on the mature predator for food. Therefore, in this paper, we consider the patches that are created by predator territories and study the dynamics of a predator–prey system with stage-structure on predators and impulsive diffusion of prey between predator territories.

The organisation of this paper is as follows. Model development is discussed in Section 2, and preliminary lemmas are established in connection with the global behaviour of the system in Section 3. In Section 4, using a comparison theorem, the global attractivity of a predator extinction periodic solution and the permanence of the system are investigated. A numerical simulation is given and discussed in Section 5, and the last section presents the conclusion.

## 2. Model formulation

Several stage-structured single species and predator–prey models have been analysed by many researchers (Jiao and Chen, 2008; Aiello and Freedman, 1990; Murray, 1989; Jiao et al., 2008; Wang and Chen, 1997). Aiello and Freedman (1990) studied the stage-structured single-species model described by the system

$$\begin{aligned} \frac{dx(t)}{dt} &= \beta y(t) - rx(t) - \beta e^{-r\tau} y(t - \tau), \\ \frac{dy(t)}{dt} &= \beta e^{-r\tau} y(t - \tau) - \eta y^2(t), \end{aligned} \quad (2.1)$$

where  $x(t)$ ,  $y(t)$  represents the immature and mature population densities, respectively, at any time  $t > 0$ . Here they let  $\tau$  be the maturation time delay, and they let  $\beta$  and  $r$  be the growth and death rates, respectively, of the immature population. For intra-species interaction,  $\eta$  is the death rate of the mature population. Further, the term  $\beta e^{-r\tau} y(t - \tau)$  represents the number of immature population who were born at time  $t - \tau$  and survive at time  $t$ , and this term will represent the transformation from the immature to the mature stage.

Another interesting single species model with impulsive diffusion was considered by Hui and Chen (2005); their system is given by

$$\begin{aligned} \left. \begin{aligned} \frac{dx_1(t)}{dt} &= x_1(t)(a_1 - b_1 x_1(t)), \\ \frac{dx_2(t)}{dt} &= x_2(t)(a_2 - b_2 x_2(t)), \end{aligned} \right\} t \neq n\tau, \\ \left. \begin{aligned} \Delta x_1(t) &= d_1(x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2(x_1(t) - x_2(t)), \end{aligned} \right\} t = n\tau, \quad n = 1, 2, \dots, \end{aligned} \quad (2.2)$$

where they supposed that the system was composed of two patches connected by movement of population  $x_i$  between  $i$ th patch,  $i = 1, 2$ . The parameters  $a_i$ ,  $a_i/b_i$  and  $d_i$ ,  $i = 1, 2$  are the intrinsic growth rates, carrying capacities and dispersal rates, respectively, in the  $i$ th patch. Moreover, the pulse diffusion occurs every  $\tau$  time unit ( $\tau$  is the positive constant representing the period). The system evolves from its initial state without being further affected by diffusion until the next pulse appears, and the net exchange from the  $j$ th patch to  $i$ th patch is proportional to the difference  $x_j - x_i$  in population densities. Here  $\Delta x_i = x_i(n\tau^+) - x_i(n\tau^-)$ , where  $x_i(n\tau^+)$  represents the density of population in the  $i$ th patch immediately after the  $n$ th diffusion pulse, and  $x_i(n\tau^-)$  represents the density of population in the  $i$ th patch before the  $n$ th diffusion pulse at time  $t = n\tau$ .

Furthermore, Jiao et al. (2009) considered a stage-structured predator–prey model with prey-impulsive diffusion between two patches in which the predator is available only in the second patch, which can be represented as follows:

$$\begin{aligned} \left. \begin{aligned} \frac{dx_1(t)}{dt} &= x_1(t)(a_1 - b_1 x_1(t)), \\ \frac{dx_2(t)}{dt} &= x_2(t)(a_2 - b_2 x_2(t)) - \beta x_2(t) y_2(t), \\ \frac{dy_1(t)}{dt} &= \alpha y_2(t) - \alpha e^{-\omega \tau_1} y_2(t - \tau_1) - \omega y_1(t), \\ \frac{dy_2(t)}{dt} &= \alpha e^{-\omega \tau_1} y_2(t - \tau_1) + k \beta x_2(t) y_2(t) - d y_2(t), \end{aligned} \right\} t \neq n\tau, \\ \left. \begin{aligned} \Delta x_1(t) &= d_1(x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2(x_1(t) - x_2(t)), \\ \Delta y_1(t) &= 0, \\ \Delta y_2(t) &= 0, \end{aligned} \right\} t = n\tau, \quad n = 1, 2, \dots \end{aligned} \quad (2.3)$$

Here, the authors assumed that the system is composed of two patches connected by diffusion and occupied by a single prey species;  $x_i(t)$ ,  $i = 1, 2$  are the densities in the  $i$ th patch at time  $t$ . The densities of immature and mature predators in the second patch at time  $t$  are denoted by  $y_1(t)$  and  $y_2(t)$ , respectively. The parameter  $\tau_1$  represents the constant time to maturity,  $\beta$  is the capture rate of a mature predator,  $k$  is the rate of conversion, and  $\omega$  and  $d$  are the natural death rates of immature and mature predators, respectively. Motivated by the systems discussed in (2.1)–(2.3), we propose in this paper a predator–prey model with the following assumptions:

- (A1) The patches are created by predator territories (i.e., each predator species is restricted to a particular patch).
- (A2) The prey population grows logistically with different growth rates in different patches due to resource availability in different patches, and moreover, the prey population has impulsive movement between the patches.
- (A3) The predator population has two distinct stages, namely immature and mature stages. The immature predator population requires a constant time to become mature.
- (A4) Mature predators can harvest the prey population with different predation rates in different patches.
- (A5) Immature predators only feed from mature predators and do not have the ability to reproduce.

These assumptions are reasonable for a number of mammals whose immature populations are raised by their parents and whose reproduction and attacking rates for prey are negligible. Therefore, our proposed mathematical model is

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