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Effects of market price on the dynamics of a spatial fishery model: Over-exploited fishery/traditional fishery

Pierre Auger[a], Rachid Mchich[b,*], Nadia Raïssi[c], Bob W. Kooi[d]

[a] IRD, UMI 209 UMMISCO, Centre de Recherche d'Ile de France, 32 Av. Henri Varagnat, 93143 Bondy Cedex, France
[b] Ecole Nationale de Commerce et de Gestion, B.P. 1255, 90000 Tanger, Morocco
[c] Equipe E.I.M.A., Faculté des Sciences, Université Ibn Tofail, B.P. 133, Kénitra, Morocco
[d] Dept. of Theoretical Biology, Faculty of Earth and Life Sciences, Vrije Universiteit, De Boelelaan 1085, 1081 HV Amsterdam, The Netherlands

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ABSTRACT

We present a dynamical model of a spatial fishery describing the time evolution of the fish stock, the fishing effort and the market price of the resource. The market price is fixed by the gap between the supply and the demand. Assuming two time scales, we use "aggregation of variables methods" in order to derive a reduced model governing fish density and fishing effort at a slow time scale. The bifurcation analysis of the reduced model is performed. According to parameters values, three main cases can occur: (i) a stable fishery free equilibrium, (ii) a stable persistent fishery equilibrium and (iii) coexistence of three strictly positive equilibria, two of them being stable separated by a saddle. In this last case, a stable equilibrium corresponds to a traditional fishery with large fish stock, small fishing effort and small market price. The second stable one corresponds to over-exploitation of the resource with small fish stock, large fishing effort and large market price.

1. Introduction

Bioeconomics has considered the dynamics of renewable resources mainly from the point of view of control theory (Clark, 1990), focusing on the existence of a Maximum Sustainable Yield. In several works, the price of the resource is constant or is a function of variables and the catch (Smith, 1968, 1969; Barbier et al., 2002). We also refer to a classical textbook in Mathematical Biology (Murray, 1993). At our knowledge, little attention has been paid to model the fishery dynamics as a dynamical system in which the market price of the resource is also a variable. However, the validity of the assumption of a constant fish price seems limited to small scale fishery. Furthermore, according to classical economic theory (Walras, 1874), the price variation depends on the gap between a demand function (that is the quantity of fish purchased by consumers) and the supply which is no more than the catch. This suggests to consider that the price is not a constant but a variable. The aim of this manuscript is thus to investigate the effects of price variation on the dynamics of the fishery and its asymptotic behaviour. We particularly look for existence of a desirable situation such as a sustainable fishery in the sense that the resource can be exploited in a durable way, but without any risk of extinction of the fish stock.

Therefore, the main point of this model is to consider a model of a fishery with an extra equation for the variation of the market price. Here, we have made two main assumptions. A first assumption considers a linear demand function such as in Lafrance (1985). Such a linear function is assumed to be a decreasing function of market price with a maximum value A when the price is equal to zero. Furthermore, we assume that the adjustment of the market price occurs rapidly with respect to fish growth, investment (new boats entering the fishery) and capture. This last assumption allows to consider a slow-fast model in which two time scales are involved leading to the possibility to use time scale separation methods to derive a reduced simplified model at a slow time scale. This simplified model in dimension 2 appears to be much simpler to handle than the original complete model in dimension 3. However, we want to mention here that our results can be extended to the general case of a price which does not vary rapidly, as will be shown in the section devoted to numerical analysis.

Our model is presented in the context of spatial fishery but it could be considered as a general model of resource exploitation with price variation. This work follows earlier contributions of simple fishery models (Mchich et al., 2002, 2006) and extends it to the case of price variation of the resource. The manuscript is

* *Corresponding author.* Tel.: +212 63 03 62 38; fax: +212 39 31 34 93.
E-mail address: racmchich@yahoo.com (R. Mchich).

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organized as follows. In Section 2, we present the mathematical model for the fishery with a price equation. In Section 3, we take advantage of the existence of two time scales to obtain a reduced model, "the aggregated model", governing the fishing effort and the resource density. We proceed to the analysis of this reduced model by use of bifurcation analysis and we show that different cases can occur. In Section 4, we give the bifurcation analysis of the aggregated model. Section 5 is devoted to the interpretation and to the discussion of the results, particularly of the case of multi-stability, and to the discussion of several perspectives for this work.

2. Presentation of the fishery mathematical model with price equation

Earlier contributions considered a system of two equations, a first one governing the time variation of the resource $n(t)$ with mass in pounds or a density with mass per unit of area and a second equation for the fishing effort $E(t)$ or more generally (in case of another resource than fish), the number of firms (assumed to be homogeneous) involved in the exploitation of the resource at time t (Smith, 1968, 1969; Barbier et al., 2002). Usually, a first equation describes the evolution of the fish stock which grows naturally and is harvested by the fishing fleet. A second equation describes the evolution of the fishing effort depending on the difference between the benefit and the cost of the fishery fixing the investment of revenues for the fishing effort. Some models considered time continuous models (Smith, 1968, 1969) as well as time discrete versions (Barbier et al., 2002). Our model is a time continuous version of the model proposed by Barbier et al. (2002), page 349:

$$\begin{cases} \dfrac{dn}{dt} = f(n) - h(n,E) \\ \dfrac{dE}{dt} = \varphi(ph(n,E) - cE) \end{cases} \tag{1}$$

where function $f(n)$ is the natural growth function of the resource, $h(n,E)$ is the harvesting function depending on the resource and the fishing effort, c is the cost per unit of fishing effort due to fuel costs, taxes, salaries and so on. φ is an adjustment positive coefficient depending on the fishery. p is the landed fish price per unit of harvested fish assumed to remain constant. Classically, in predator–prey models, the predation (harvesting) function is usually written as follows: $h(n,E) = g(n,E)E$. The function $g(n,E)$ is called the functional response, i.e. the amount of resource captured per unit of time and per unit of predator (in the context of fishery, per unit of fishing effort). In fishery modelling, it is usual to assume a Schaefer function for the harvesting function, i.e. $g(n,E) = qn$ where q is a positive constant which is called the catchability (the capturability in the context of fishery) (Schaefer, 1957). In this case, the functional response depends linearly on the resource and is also classically mentioned in ecological modelling as the type I functional response or the law of mass action. In the context of predator–prey models, many other functional responses have been considered such as the Holling type II, the type III, the ratio-dependent functional responses and more, see Bazykin (1998) and Edelstein-Keshet (1998). In the context of fishery modelling, the Schaefer function is most commonly considered. However, we mention another example, in Smith (1969), Section 2, where the functional response is rather similar to a Holling type II functional response but with a significant difference due to the presence of the fishing effort in its denominator leading to a functional response which is unusual in Ecology.

If we consider as usual a logistic natural growth function, $f(n) = rn(1-(n/K))$, where r is the growth rate of the resource and K its carrying capacity and a Schaefer harvesting function as usual in fishery modelling, the model reads:

$$\begin{cases} \dfrac{dn}{dt} = rn\left(1 - \dfrac{n}{K}\right) - qnE \\ \dfrac{dE}{dt} = \varphi E(pqn - c) \end{cases} \tag{2}$$

The model with the logistic growth in Smith (1968, 1969) is related to a logistic growth with Allee strong effects where there is a positive threshold where the population is not viable because of vulnerability to disease, parasites, predators, or to inadequate fecundity.

2.1. A spatial fishery model with fast migration

In Mchich et al. (2002, 2006) we considered a spatial fishery with two fishing areas. In these models, we assumed that the fishing areas remain close and that fish stock as well as fishing boats could move rapidly.

The first two equations in (3a) describe the evolution of two fish stocks which are dispatched on two fishing zones on which they are harvested by two fleets. The two remaining equations in (3a) describe the evolution of the fishing efforts. Each of the equations in (3a) contains two parts: a first one describing migration between two fishing zones at a fast time scale and a second one describing natural growth and catch-ability for fish stocks and the investment of revenues for the fishing efforts. The second part of any equation holds at a slow time scale.

According to these assumptions, the complete system, at the fast time scale $\tau = \varepsilon t$ (with respect to the slow time scale t; $\varepsilon \ll 1$), reads as follows:

$$\begin{cases} \dfrac{dn_1}{d\tau} = (kn_2 - k'n_1) + \varepsilon\left[r_1 n_1\left(1 - \dfrac{n_1}{K_1}\right) - q_1 n_1 E_1\right] \\ \dfrac{dn_2}{d\tau} = (k'n_1 - kn_2) + \varepsilon\left[r_2 n_2\left(1 - \dfrac{n_2}{K_2}\right) - q_2 n_2 E_2\right] \\ \dfrac{dE_1}{d\tau} = [mE_2 - m'E_1] + \varepsilon E_1(pq_1 n_1 - c_1) \\ \dfrac{dE_2}{d\tau} = [m'E_1 - mE_2] + \varepsilon E_2(pq_2 n_2 - c_2) \end{cases} \tag{3a}$$

where r_i (for $i = 1,2$) represents the intrinsic growth rate of the stock n_i. Patches have distinct characteristics, so we suppose that parameters r_1 and r_2 are different such as carrying capacities, K_i (for $i = 1,2$) and q_i (for $i = 1,2$) which are catch-ability coefficients of the fleet on patch i. The catch-ability is supposed to be constant.

2.2. Variable price of the resource

In most contributions, the price of the resource p is assumed to be either constant (leading to a very classical prey–predator model (Bazykin, 1998; Edelstein-Keshet, 1998)). In some other cases, the price is not constant and is assumed to depend on variables such as the harvesting function, the resource, or the fishing effort (Barbier et al., 2002). However, the outcome of the model may strongly depend on the choice of this price function.

More generally, one can consider a new variable, $p(t)$, the price of a stock unit at the time t. Therefore, it is necessary to add a fifth equation to the previous model governing the variation of the price with time. In the next model, with respect to these previous contributions, we only add an equation for the price which is assumed to vary rapidly such as in previous sections.

The simplest assumption is that the price varies according to the difference between the demand, $D(p)$, and the supply which is simply the catch. Furthermore we assume that the price of the