Ecological Informatics 35 (2016) 19–28

Contents lists available at ScienceDirect

Ecological Informatics

journal homepage: www.elsevier.com/locate/ecolinf

Effect of salinity and fish predation on zooplankton dynamics in Hooghly–Matla estuarine system, India

Ujjwal Roy[a], Sahabuddin Sarwardi[b,*], N.C. Majee[a], Santanu Ray[c]

[a] Department of Mathematics, Jadavpur University Kolkata - 700 032 West Bengal, India
[b] Department of Mathematics, Aliah University, IIA/27, New Town, Kolkata - 700 156, West Bengal, India
[c] Ecological Modelling Laboratory, Department of Zoology, Visva- Bharati University, Santiniketan - 731 235, West Bengal, India

ARTICLE INFO

Article history:
Received 1 January 2016
Received in revised form 11 May 2016
Accepted 8 July 2016
Available online 19 July 2016

MSC Classification:
92D25
92D30
92D40

Keywords:
PZF model
Equilibria
Local stability
Global stability
Hopf bifurcation
Numerical simulation

ABSTRACT

The Hooghly–Matla estuarine complex is the unique estuarine system of the world. Nutrient from the litterfall enrich the adjacent estuary through tidal influence which in turn regulate the phytoplankton, zooplankton and fish population dynamics. Environmental factors regulate the biotic components of the system, among which salinity plays a leading role in the regulation of phytoplankton, zooplankton and fish dynamics of the estuary. In this article, a *PZF* model is considered with Holling type-II response function. The present model considers salinity based equations on plankton dynamics of the estuary. The interior equilibrium is considered as the most important equilibrium state of this model. The model equations are solved both analytically and numerically using the real data base of Hooghly–Matla estuarine system. The essential mathematical features of the present model have been analyzed thorough local and global stability and the bifurcations arising in some selected situations. A combination of set of values of the salinity of the estuary are identified that helped to determine the sustenance of fish population in the system. The ranges of salinity under which the system undergoes Hopf bifurcation are determined. Numerical illustrations are performed in order to validate the applicability of the model under consideration.

1. Introduction

The delta of Hooghly-Matla estuarine system is networked by seven major rivers along with their tributaries and creeks. This deltaic system harbours luxuriant mangroves and constitutes Sundarban mangrove ecosystem. The mangroves are the major resources of detritus and nutrients to the adjacent estuary that make up favourable habitat for the growth of shell fish and fin fish (cf. Mandal et al., 2012). This estuary act as route and refuge areas for a variety of migratory fish species. The estuary supports 53 species of pelagic fish belonging to 27 families and 124 species under 49 families of demersal fish (cf. Hussain and Acharya, 1994). Fishery in this estuarine water contributes a part in the economy of the state of West Bengal, India. The organic matter that passes from litterfall to the adjacent estuary supports both the grazing and detritus food chains of this lotic system.

Among the chemical components studied so far, salinity plays a crucial role in the abundance (cf. Bhunia, 1979) and dynamics of zooplankton of the estuary (cf. Ghosh, 2001; Ketchum, 1951). Because, this community lies in the middle of grazing food chain: phytoplankton–zooplankton–fish (*PZF*). In addition, perturbation to this trophic level may trigger imbalance in the food chain which in turn affect the phytoplankton and fish community. The dynamics of salinity is season dependent. During monsoon and early post monsoon, huge fresh water enters in the estuary from the upstream resulting in the lowering of salinity. In the pre-monsoon, fresh water runoff from the upstream

* Corresponding author.
E-mail addresses: ujjwal2ju@gmail.com (U. Roy), s.sarwardi.math@aliah.ac.in (S. Sarwardi), sray@visva-bharati.ac.in (S. Ray).

http://dx.doi.org/10.1016/j.ecoinf.2016.07.002
1574-9541/

becomes very less and due to tidal influence of the adjacent Bay of Bengal, the salinity increases. Throughout the year, a gradient of salinity is observed between upstream and downstream area of the estuary (cf. Mandal et al., 2012).

The abundance of different species of zooplankton varies according to the salinity of the estuary throughout the year, as a result the grazing rate also changes with seasons. In *PZF* system, the grazing rate of zooplankton is one of the most sensitive parameter in Hooghly-Matla estuarine system as the dynamics of zooplankton also depends on the lower trophic level of phytoplankton as well as fish population (cf. Mandal et al., 2012). Moreover, fish predation exhibits top-down effect on zooplankton community. Therefore, in the *PZF* system, two important parameters that shape up the zooplankton dynamics of the estuary are salinity dependent grazing rate of zooplankton and fish predation rate (cf. Dube et al., 2010).

There have been only a few models based on the effects of salinity on plankton dynamics. Few studies have been done in the relationship between salinity levels to the types of species that can occur in algal blooms (cf. Griffin et al., 2001; Marcarelli et al., 2006; Quinlan and Phlips, 2007). Many *PZF* model are constructed on estuarine system (cf. Ray et al., 2001a,b; Cottingham et al., 2004; Dube and Jayaraman, 2008; Dube et al., 2010) and top-down effect (cf. Morozov et al., 2005; Irigoien et al., 2005; Calbet and Saiz, 2005) but none of the models have considered the salinity dependent grazing rate of zooplankton which plays an important role in the zooplankton dynamics of estuary. The present account deals with a *PZF* model, where salinity dependent grazing rate of zooplankton is taken into consideration along with the variation of upstream and downstream salinity.

1.1. Mathematical model formulation

Let P, Z, and F denote the populations of phytoplankton, zooplankton and fish respectively. In this present *PZF* model, light and temperature dependant photosynthesis rate of phytoplankton and salinity induced grazing rate of zooplankton have been incorporated into the model proposed by Mandal et al. (2012). The modified model under consideration is as follows:

$$\begin{cases} \frac{dP}{dt} = m_1 P\left(1-\frac{P}{k_P}\right) - g_s\frac{PZ}{P+k_Z} \\ \frac{dZ}{dt} = ag_s\frac{PZ}{P+k_Z} - g_f\frac{ZF}{Z+k_F} - m_2 Z \\ \frac{dF}{dt} = g_f\frac{ZF}{Z+k_F} - m_3 F. \end{cases} \tag{1}$$

The salinity induced grazing of zooplankton is $g_s = \delta g_Z$, where $\delta = \frac{s_u}{s_u - s_d}$, the dilution factor and m_1, s_d, s_u, g_Z, k_Z, k_P are net growth rate of phytoplankton, downstream salinity, upstream salinity, grazing rate of zooplankton, half saturation constant of zooplankton on phytoplankton grazing by zooplankton and half saturation constant grazing by phytoplankton respectively. Since estuary is a transition zone of river and sea, so there is always fluctuation of salinity throughout the year, which is due to dilution by upstream river water and/or mixing by downstream sea tidal water. δ is calculated by following the equation of Ketchum (1951). Besides grazing the abundance of zooplankton is also dependent on loss due to $E_{zoo} = ZE_{zo}$, respiration $R_{zoo} = Zr_{zo}$, fish predation $F_p = Zr_{rf}$, and mortality $M_{zoo} = ZM_Z$, R_{zoo} is governed by r_{zo}.

The net mortality rate of zooplankton is $m_2 = (E_{zo} + r_{zo} + r_{fp} + M_Z)$. The abundance of F is governed by many processes in the estuary. Fish predation F_p on Z follows Michaelis–Menten kinetics (Holling type II) which enriches the fish pool of the estuarine system. F_p depends on k_Z and g_f. F population is reduced by mortality rate of fish M_f, respiration Fr_f, harvest by fishing FH_{cf} and excretion of fish FE_f. FH_f are controlled by R_f and H_f respectively. R_f, respiration rate of fish and H_f, harvest rate of carnivorous fish respectively.

The net mortality rate of fish is $m_3 = (E_f + M_f + r_f + H_f)$, where r_{fp}, r_{zo} and M_{rz} are fish predation rate, zooplankton respiratory rate and mortality rate respectively.

1.2. Existence and positive invariance

Letting $X \equiv (P,Z,F)^T, f : \mathbb{R}^3 \to \mathbb{R}^3, f = (f_1,f_2,f_3)^T$, the system Eq. (1) can be rewritten as $\dot{X} = f(X)$. Here $f_i \in C^\infty(\mathbb{R})$ for $i = 1,2,3$, where $f_1 = m_1 P\left(1-\frac{P}{k_P}\right) - g_s\frac{PZ}{P+k_Z}, f_2 = ag_s\frac{PZ}{P+k_Z} - m_2 Z - \frac{ZF}{Z+k_F}, f_3 = g_f\frac{ZF}{Z+k_F} - m_3 F$. Since the vector function f is a smooth function of the variables P, Z and F in the positive octant $\Omega^0 = \{(P,Z,F); P > 0, Z > 0, F > 0\}$, the local existence and uniqueness of the system hold.

1.3. Boundedness of the system

Boundedness of a system guarantees its biological validity. The following theorem establishes the uniform boundedness of the system Eq. (1).

Theorem 1. *All the solutions of the system Eq. (1) which start in $\mathbb{R}^3_+$ are uniformly bounded.*

Proof. Let $(P(t), Z(t), F(t))$ be any solution of the system with positive initial conditions. From the real ecological field study one can consider $\max\{m_2, m_3\} < 1$.

Now let us we define the function $X = aP + Z + F$. The time derivative of X gives

$$\begin{aligned} \frac{dX}{dt} &= a\frac{dP}{dt} + \frac{dZ}{dx} + \frac{dF}{dt}, \\ &= am_1 P\left(1-\frac{P}{k_P}\right) - ag_s\frac{PZ}{P+k_Z} + ag_s\frac{PZ}{P+k_Z} - g_f\frac{ZF}{Z+k_F} - m_2 Z + g_f\frac{ZF}{Z+k_F} - m_3 F \\ &= am_1 P\left(1-\frac{P}{k_P}\right) - m_2 Z - m_3 F. \end{aligned} \tag{2}$$