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Analogy between Sudoku and the multi-scale integrated analysis of societal metabolism

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ABSTRACT

This paper illustrates the analogy between Sudoku and the multi-scale integrated analysis of societal and ecosystem metabolism (MuSIASEM). MuSIASEM is a transdisciplinary approach aimed at checking the viability, desirability and feasibility of actual and projected metabolic patterns of society. Building on Georgescu-Roegen's flow-fund model, it defines what the system *is* in terms of fund elements (human activity, managed land, power capacity), and what it *does* in terms of flow elements (energy, food, water, monetary flows). The accounting method of MuSIASEM generates a multi-scale, multi-dimensional representation of flow and fund elements that shares essential features with Sudoku, including mutual information and impredicativity between bottom–up and top–down causality. Data organization employed in MuSIASEM reveals the internal and external constraints that operate on the societal metabolic pattern and that determine its viability and feasibility domain. The dynamic equilibrium between the hypercyclic and dissipative macro-compartments of society expresses itself as a regional constraint on the viability of the metabolic pattern. The MuSIASEM approach is illustrated with case studies including the energy metabolism of Spain and the metabolic pattern of Mauritius.

1. Introduction

Concerns about the sustainability of human development have given rise to a series of composite indicators for use in governance, such as the Environmental Sustainability Index or the Ecological Footprint (Singh et al., 2012). Composite indicators have the goal of collapsing a complex information space encompassing multidimensional issues into a single numerical index. This approach has obvious advantages for policy makers but, from a scientific point of view, usually represents a brutal simplification of reality (Böhringer and Jochem, 2007).

The multi-scale integrated analysis of societal and ecosystem metabolism (MuSIASEM) proposes a radically different procedure to handle quantitative data belonging to non-equivalent descriptive domains. MuSIASEM is rooted in complex systems theory and rather than simplifying the information space represented by societal metabolism, it focuses on revealing the complex interrelations between structural and functional elements of society across various hierarchical levels of organization. In doing so, we find a strong similarity with the popular Sudoku game. Indeed, despite its simplicity, Sudoku exemplifies two key characteristics of complex adaptive systems: the existence of mutual information and impredicativity between bottom–up and top–down causality. We use the Sudoku analogy to illustrate the logic behind our approach and to show that the evolution of complex adaptive systems, such as societies, can be studied in two non-equivalent ways: bottom–up (studying local-scale characteristics to explain emergent properties at the larger scale) and top–down (studying large-scale characteristics to analyse the constraints on lower-level processes). The analogy with Sudoku becomes especially evident when MuSIASEM is used for simulation purposes.

For those unfamiliar with Sudoku, we first briefly introduce the game and its features. We then introduce MuSIASEM, explaining what it is and how it works. Following we draw the analogy with Sudoku to underscore the essence and importance of systems thinking in our approach. Along the way, we use practical examples from the metabolic pattern of Spain and Mauritius, to explain our arguments.

2. Sudoku: a metaphor for systems thinking

Sudoku is a logic-based number placement puzzle. The game of Sudoku provides a concrete example of how to generate a complex information space by combining constraints defined at different scales. In Sudoku the objective is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 sub-grids (regions or blocks) that compose the grid, contains all of the digits from 1 to 9 (see Fig. 1). This combination of vertical, horizontal and regional constraints generates 'mutual information' within the information space of the Sudoku grid. As a result, each time we introduce a number in the grid we dramatically reduce the degree of freedom for the remaining numbers to be entered. In this way, the information accumulated

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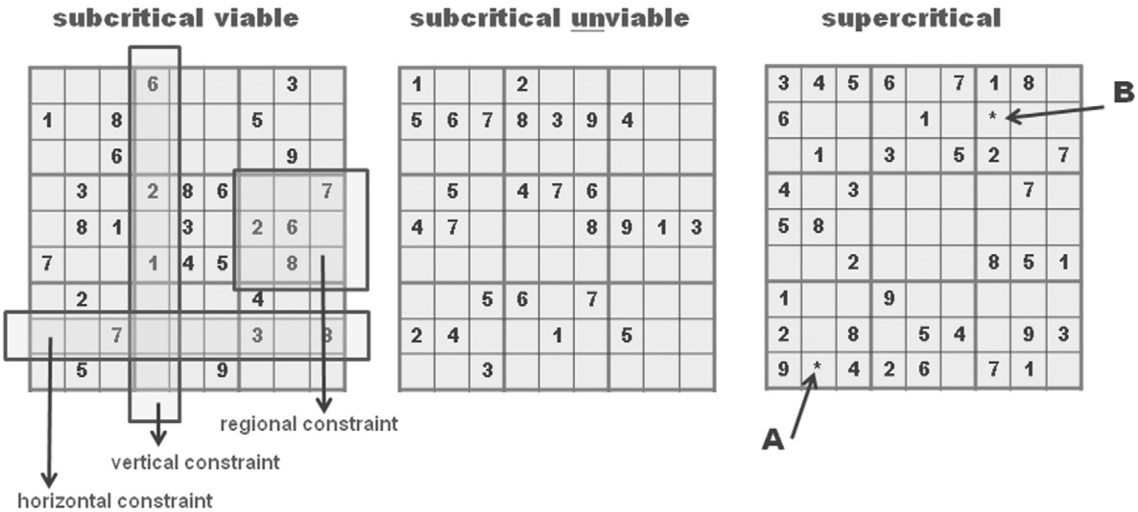

Fig. 1. Subcritical and viable Sudoku, subcritical and unviable Sudoku, and supercritical Sudoku (Prof. Shin-ichi Nakayama, University of Tokushima, personal communication).

in the Sudoku reduces the option space of viable numerical patterns. Indeed, the numbers entered in individual cells of the grid (local scale) constrain the overall numerical pattern (bottom–up causation), whereas the partial overall pattern already present constrains the entry of new numbers into the grid (top–down causation).

As illustrated in Fig. 1, depending on the number of solutions of the Sudoku puzzle, we can distinguish the following three cases:

(1) Subcritical, viable Sudoku — Playing this Sudoku we will find that the numbers already printed in the grid (clues) limit the possible patterns for the remaining numbers to a point that the Sudoku has a unique solution. Under this particular condition of subcriticality, the mutual information provided by the integrated set of constraints (the rules of the game) and the given numbers (accumulated history), allows us to identify the missing numbers in the given pattern. This is the Sudoku players are familiar with;
(2) Subcritical, unviable Sudoku — Playing this Sudoku we will quickly realize that it is simply impossible to fill all the cells of the grid. In this Sudoku, there is an incompatibility between the numbers already given and the rules of the game.
(3) Supercritical Sudoku — Playing this Sudoku we sooner or later will discover that it is not a deterministic system. Different players may obtain different solutions. Indeed, the supercritical Sudoku illustrated in Fig. 1 may yield four different solutions. However, as soon as the player enters new valid numbers in the Sudoku grid, the further accumulation of mutual information increases its degree of determinedness. For example, if A = 5 is entered, then only 3 solutions are left; if B = 3 is entered next, then the Sudoku becomes subcritical admitting only one solution.

Sudoku is extremely useful to explain the difference between feasibility and viability. The constraints determined by the grid format of the Sudoku and the rules of the game together define the feasibility domain for the pattern of numbers that can be entered in the columns, rows, and sub-grids. Note that this definition of feasibility does *not* depend on the clues given. As soon as numbers are entered into the grid, a new set of constraints is generated by the accumulation of history. Indeed, this input of information adds additional constraints of a new type to the original set and reduces the feasibility space into a smaller viability space. Clearly, accumulation of internal constraints may lead to lack of a solution if numbers are placed in an invalid position generating a subcritical unviable Sudoku.

We generally only play Sudoku puzzles that are purposely designed to have a unique solution. If we were to entirely self-organize our Sudoku from scratch (no numbers given) without using a computer algorithm, the tentative filling of the grid would demand a tremendous effort, especially without any means to detect in an early stage whether or not we are trying to solve an unsolvable Sudoku! Unfortunately though, the latter situation appears to be exactly the predicament of sustainability science when trying to check the feasibility, viability and desirability of possible scenarios of societal metabolism. Indeed, the *supercritical* Sudoku provides an apt analogy for the analysis of the societal metabolic pattern.

3. Multi-scale Integrated Analysis of Societal and Ecosystem Metabolism

Multi-scale Integrated Analysis of Societal and Ecosystem Metabolism (MuSIASEM) is a transdisciplinary approach to sustainability assessments that integrates quantitative information generated by distinct types of conventional models based on different dimensions and scales of analysis. It builds on concepts derived from bioeconomics and complex systems theory, such as Georgescu-Roegen's flow-fund model, multi-purpose grammars and impredicative loop analysis (Giampietro et al., 2012). The application of these concepts allows the *simultaneous* use of economic, social, demographic, biophysical, and ecological variables in the analysis of the metabolic pattern of society, even if these variables are defined within distinct descriptive domains and refer to different hierarchical scales. Given this feature, MuSIASEM provides the means to effectively analyse socio-ecological systems considering heterogeneous factors such as population dynamics, energy resources, water resources, economic variables and land-use changes at the national or sub-national level. MuSIASEM can be employed for diagnostic as well as for simulation purposes.

As diagnostic tool, the MuSIASEM accounting system is used to describe and typify the actual metabolic pattern of the socio-economic system under analysis by providing integrated information on:

1) Population, work force, technological capital, managed land, and total available land (defined as *fund elements*[1]);

[1] The terms fund and flow elements are derived from Georgescu-Roegen's flow-fund model (Georgescu-Roegen, 1971). In Georgescu-Roegen's analytical model *flow elements* are represented by inputs (e.g., food, energy and mineral inputs) and wastes (e.g., garbage, GHG and other pollutants), *fund elements* are represented by structural elements (e.g., human beings, land, technical capital) that compose the functional compartments (e.g., economic sectors) of the system. *Fund elements* have to be preserved (reproduced) in the medium term and improved in the long term for the system to survive. This means that their identity has to remain the same over the time scale of the analysis (e.g., hours, day, years, or decades). On the contrary, *flow elements* represent inputs and outputs required to reproduce these fund elements, such as energy, material, water, monetary flows. By definition flows have to either appear (outputs) or disappear (inputs) over the duration of the analysis.