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Short communication

Odum–Tennenbaum–Brown calculus vs emergy and co-emergy analysis

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ABSTRACT

In a recent paper Tennenbaum introduced a new method of calculating emergy that requires only ordinary (i.e. linear) algebra. We prove on a simple example with one feedback and one split that ordinary algebra as developed by Tennenbaum in his paper is not sufficient to tackle the problem of emergy analysis. In particular, we point out the problem of enumerating pathways which are relevant for emergy analysis, i.e. which avoid the double counting problem of feedbacks. Hence, the emergy co-emergy analysis cannot work at least for energy system diagram with splits and feedbacks. Le Corre and Truffet have already proved that the emergy path-finding problem deals with idempotent (thus non-linear) algebra.

1. Introduction

Emergy is a concept defined as the total solar equivalent energy/exergy of one form that was used up directly or indirectly in the work of making a product or a service. It means that emergy analysis cannot be validated by experimentation except for some trivial cases. Thus, as in e.g. theoretical physics, emergy analysis of a given system requires at least a rigorous mathematical framework. This mathematical framework is represented by a formula or a set of axioms/rules which allows us to compute the emergy.

In Odum (1996, pp. 99–101) the *Track summing method* developed by Tennenbaum (1988) was presented on an example (see Odum, 1996, p. 101). This method is based on the computation of the emergy pathways from a source to the output of a node of an energy system diagram at which the emergy has to be calculated. Based on these works, it seems that Brown and Herendeen (1996) were the first to propose the following four rules of emergy computation named *emergy algebra*:

(R1). All source emergy to a process is assigned to the processes's output(s).

(R2). By-products from a process have the total emergy assigned to each pathway.

(R3). When a pathway splits, the emergy is assigned to each 'leg' of the split based on its percent of total energy flow on the pathway.

(R4). Emergy within a system of interconnected components cannot be counted twice.

(R4.1). Emergy in feedbacks cannot be double counted;

(R4.2). By-products, when reunited, cannot be added to equal a sum greater than the source emergy from which they were derived.

A consequence of emergy algebra is that emergy calculus does not obey Kirchoff-like circuit law. Thus, the aims of this short communication are as follows:

- Clarify the Odum–Tennenbaum–Brown approach
- Show that emergy co-emergy analysis (see Tennenbaum, 2014) of systems with splits and feedbacks based on linear algebra cannot be exact. And to give the correct values of emergy in Appendix C on the example of Tennenbaum (2014, subsection 2.2.1).

2. Linear algebra fails: an example

2.1. Odum–Tennenbaum–Brown calculus vs Tennenbaum linear algebra approach

Let us consider the example of Brown (see Brown and Herendeen, 1996, Figure 8b, p. 226). See also Fig. 1 which represents the fraction of emergy flowing on each arc. In this network there are two sources $s = 400$ and $s' = 100$, one feedback and one

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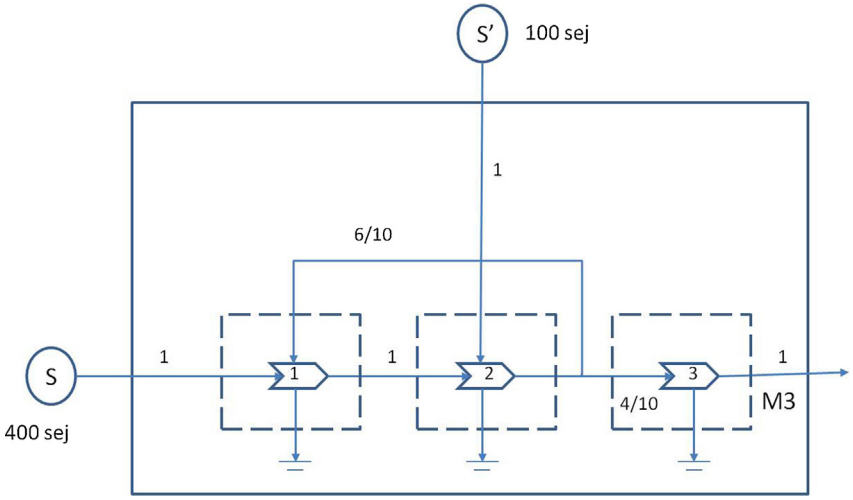

Fig. 1. Network with split at node 2.

split at node 2. The fraction of emergy which returns to node 1 is 0.6 and the fraction of emergy which goes to node 3 is 0.4.

By applying Odum–Tennenbaum–Brown calculus the emergy at output of unity 3, say M_3 using Tennenbaum (2014) notations, is:

$$M_3 = 400 \times \underbrace{0.4}_{(a)} + 100 \times \underbrace{0.4}_{(b)} = 200, \tag{1}$$

where, according to rule (R4.1) and track summing method, (a) is the weight of the path $[s\,;1][1\,;2][2\,;3]$ and (b) is the weight of the path $[s'\,;2][2\,;3]$. These two paths are the only ones which are relevant to Odum–Tennenbaum–Brown calculus.

According to emergy co-emergy analysis (see Tennenbaum, 2014) one has to solve the following system:

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} (1-f_{1,1}) & -f_{2,1} & 0 \\ -f_{1,2} & (1-f_{2,2}) & 0 \\ -f_{1,3} & -f_{2,3} & 1 \end{bmatrix} \begin{bmatrix} C_{1,[3]} \\ C_{2,[3]} \\ M_3 \end{bmatrix}$$

with $E_1 = 400$, $E_2 = 100$ and $E_3 = 0$. And:

$$F = \begin{bmatrix} f_{1,1} & f_{1,2} & f_{1,3} \\ f_{2,1} & f_{2,3} & f_{2,3} \\ f_{3,1} & f_{3,2} & f_{3,3} \end{bmatrix} = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.6 & 0.0 & 0.4 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}.$$

By applying Tennenbaum (2014, formula (7)) one has:

$$\begin{bmatrix} C_{1,[3]} \\ C_{2,[3]} \\ M_3 \end{bmatrix} = \begin{bmatrix} 1.0 & -0.6 & 0.0 \\ -1.0 & 1.0 & 0.0 \\ 0.0 & -0.4 & 1.0 \end{bmatrix}^{-1} \begin{bmatrix} 400 \\ 100 \\ 0 \end{bmatrix}$$

with

$$\begin{bmatrix} 1.0 & -0.6 & 0.0 \\ -1.0 & 1.0 & 0.0 \\ 0.0 & -0.4 & 1.0 \end{bmatrix}^{-1} = \begin{bmatrix} 2.5 & 1.5 & 0.0 \\ 2.5 & 2.5 & 0.0 \\ 1.0 & 1.0 & 1.0 \end{bmatrix}$$

And thus according to Tennenbaum (2014) we have:

$$M_3 = 400 + 100 = 500. \tag{2}$$

Moreover, let α be the fraction of emergy which returns to node 1 from 2 (see Fig. 1). Then, for all $\alpha \in]0, 1[$ we have

$$\begin{bmatrix} 1.0 & -\alpha & 0.0 \\ -1.0 & 1.0 & 0.0 \\ 0.0 & -(1-\alpha) & 1.0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 0 \\ \frac{1}{1-\alpha} & \frac{1}{1-\alpha} & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

And then:

$$\begin{bmatrix} C_{1,[3]} \\ C_{2,[3]} \\ M_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} & 0 \\ \frac{1}{1-\alpha} & \frac{1}{1-\alpha} & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 100 \\ 0 \end{bmatrix}$$

so that $M_3 = 400 + 100$ and the emergy co-emergy analysis provides a result for M_3 which is completely independent of α. This fact contradicts rule (R3).

2.2. Explanation of the difference between formulas (1) and (2)

The difference between formula (1) and formula (2) is explained as follows for $\alpha = 0.6$. But note that the explanation is of the same kind for all $\alpha \in]0, 1[$.

One remarks that the matrix $G = \begin{bmatrix} 1.0 & -0.6 & 0.0 \\ -1.0 & 1.0 & 0.0 \\ 0.0 & -0.4 & 1.0 \end{bmatrix}$ is of the form $I - A$, with I denoting the identity matrix and

$$A = \begin{bmatrix} 0.0 & 0.6 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.0 \end{bmatrix} = \begin{bmatrix} 0.0 & f_{2,1} & 0.0 \\ f_{1,2} & 0.0 & 0.0 \\ 0.0 & f_{2,3} & 0.0 \end{bmatrix}. \tag{3}$$

Noticing that A is a substochastic matrix, G^{-1} can be expanded as follows (see the proof in Appendix A):

$$G^{-1} = I + A + A^2 + A^3 + \cdots + A^n + \cdots. \tag{4}$$

G^{-1} is known as the Green function. Such a matrix naturally appears in several domains: in potential theory of Markov chains (see e.g. Revuz, 1984), in the study of the discretized heat equation (see e.g. Doob, 1959), in economics where this matrix is also known as Leontief/Ghosh inverse (see e.g. Leontief, 1973; Oosterhaven, 1996). In ecology theory it has already been observed that matrix