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Reliability of migration between habitat patches with heterogeneous ecological corridors

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ABSTRACT

Natural and designed ecological corridors are key elements for the survival of a species, as they allow the species to avoid local extinction by migrating to more suitable habitat patches. This paper studies various reliability metrics for the process of migration in a metapopulation landscape network from a critical habitat patch to destination habitat patches via perfect stepping stones and imperfect (deletable) corridors. The work presented herein generalizes earlier work on the application of reliability theory in ecology by allowing corridors to be heterogeneous (of non-identical unreliabilities). The paper is a tutorial exposition of modern reliability techniques, which formulate a problem in the Boolean domain, manipulate formulas to achieve disjointness of logically added subexpressions and retain statistical independence of logically multiplied ones, and finally reach a probability-ready expression that is directly transformed back to the probability domain. Several metrics are covered including system unreliability, life expectancy (MTTF), and component importance measures. An interesting finding is that the life expectancy of a classical landscape network is more than double that of a single corridor. Extensions to quantification of uncertainty in the above metrics and to evaluation of more sophisticated metrics of landscape connectivity are also pointed out.

1. Introduction

Landscape connectivity is a key issue for the survival of a species in its natural habitat. Tischendorf and Fahrig (2000) point out the existence of many definitions for landscape connectivity. Notable among these is the definition of Taylor et al. (1993): "the degree to which the landscape facilitates or impedes movement among resource patches" and that of With and King (1997): "the functional relationship among habitat patches owing to the spatial contagion of habitat and the movement responses of organisms to landscape structure." According to Tischendorf and Fahrig (2000), these definitions accentuate the dependence of movement on landscape structure which suggests that connectivity is species and landscape-specific. Many metrics measuring landscape connectivity have been suggested (Jordán, 2000, 2003; Goodwin and Fahrig, 2002; Jordán et al., 2003; Kindlmann and Burel, 2008; Vasas et al., 2009; Baranyi et al., 2011), but notable among these are metrics directly borrowed from reliability engineering. We concentrate herein on these metrics since "reliability theory has an amazing predictive and explanatory power with a few, very general and realistic assumptions (Gavrilov and Gavrilova, 2001)." Our work extends the seminal work of Jordán (2000), in which he considered a novel approach for applying reliability theory to a prominent problem of landscape connectivity concerning ecological-corridor design. Jordán considered a meta-population landscape graph, in which points represent patches (habitat patches or stepping stones) that are perfect and cannot be deleted, and in which edges represent corridors and can be deleted independently. He made the strong simplification of assuming that corridor deletion probabilities are equal, though he admitted that "some corridors are surely more permeable and of greater safety." He suggested further development in which different measures of "corridor permeability could be created by considering the lengths and the widths of the corridors and the level of perturbations affecting the corridors."

This paper generalizes the work in Jordán (2000) by relaxing the assumption of identical corridor reliabilities. The paper is also intended to be a brief tutorial exposition of techniques of symbolic reliability analysis applicable in the study of connectivity of ecological networks. Study of symbolic and not just numerical reliability allows one to have full information about many reliability metrics, thanks to the availability of a symbolic expression for system reliability $R(\boldsymbol{q})$ in terms of component or corridor reliabilities $\boldsymbol{q}$.

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- Substitution of numerical values for the (generally non-identical) corridor reliabilities $\boldsymbol{q}$ allows the *evaluation* of a numerical value for system reliability $R(\boldsymbol{q})$. If the component reliabilities $\boldsymbol{q}$ are known only with uncertainty i.e., if their nominal or mean values together with their variances are given, then the symbolic expression $R(\boldsymbol{q})$ allows a quantification of both the nominal value and uncertainty in R (Rushdi, 1985a).
- Allowing an explicit time dependence of the corridor reliabilities $\boldsymbol{q}$ leads to a numerical value of the system life expectancy or its mean time to failure (MTTF) (Krasich, 2009; Rushdi, 2010; Krivtsov and Yevkin, 2014). This is achieved via the numerical *integration*:

$$\text{MTTF} = \int_0^\infty R(\boldsymbol{q}(t))dt. \tag{1}$$

- Assessing the relative importance of each corridor is possible typically via *differentiation* (Barlow and Proschan, 1975; Rushdi and Al-Thubaity, 1993; Kuo and Zhu, 2012; Zhu and Kuo, 2014). For example, an importance measure for corridor m is

$$I_m = \frac{\partial R(\boldsymbol{q})}{\partial q_m} = R(\boldsymbol{q}|1_m)\text{–}R(\boldsymbol{q}|0_m), \tag{2}$$

where differentiation is obtained simply as *differencing*, i.e., by subtracting the following two *instances* or *restrictions* of $R(\boldsymbol{q})$:

$$R(\boldsymbol{q}|1_m) = R(\boldsymbol{q})]q_m = 1, \tag{3a}$$

$$R(\boldsymbol{q}|0_m) = R(\boldsymbol{q})]q_m = 0. \tag{3b}$$

Reliability analysis in this paper consists of the following three-step strategy (Bennetts, 1975, 1982; Abraham, 1979; Rushdi and Al-Khateeb, 1983; Rushdi, 1983a,b, 1984a,b; Schneeweiss, 1984, 1997; Rushdi and Goda, 1985; Rushdi and AbdulGhani, 1993; Rushdi and Ba-Rukab, 2005a,b):

- formulating the problem logically in the Boolean (switching) domain, thereby obtaining an expression for the indicator variable of system success therein,
- recasting this expression in the form of a probability-ready expression (PRE), i.e., one in which *ANDed* terms are statistically independent and *ORed* entities are disjoint,
- transforming the aforementioned expression on a one-to-one basis into the algebraic (probability) domain, thereby obtaining a symbolic expression for system reliability.

The above strategy replaces the common policy of pursuing all stages of problem formulation and manipulation in the probability domain. It allows a straightforward formulation followed by an efficient scalable manipulation. To make the paper self-contained, we include in Appendix certain useful rules for implementing this strategy. It might be useful, albeit not necessary, if the reader also consults one of the many excellent texts available on reliability engineering, such as (Kaufmann et al., 1977; Henley and Kumamoto, 1985; Barlow and Proschan, 1996; Ebeling, 1997; Trivedi, 2002; Kuo and Zuo, 2003; Rausand and Hoyland, 2004; Billinton and Allan, 2005; Misra, 2008). Applications of reliability theory to ecological and biological sciences are available in (Naeem, 1998; Gavrilov and Gavrilova, 2001; Ma, 2010). Since this paper makes an extensive use of both the conventional and variable-entered versions of the Karnaugh map, the reader is also advised to consult some of the references on it (Muroga, 1979; Roth, 1993; Roth and Kinney, 2010; Rushdi, 1983a, 1985b, 1986a, 1987, 1997, 2001; Rushdi and Al-Yahya, 2000, 2001a,b, 2002; Rushdi and Amashah, 2011; Rushdi and Albarakati, 2013; Rushdi and Alturki, 2015).

The organization of the rest of this paper is as follows. Section 2 lists our assumptions, notation and certain useful nomenclature in the ecology and reliability domains. Section 3 presents a technique for the evaluation of the unreliability of an ecological network when paths to the new habitat patches do not have edges in common. The unreliability for a small landscape network is symbolically expressed for the case of heterogeneous corridors and then twice for the case of identical corridors via (a) a purely additive formula and (b) an all-reliability formula. The unreliabilities of the overall network and its subnetworks are numerically computed and plotted versus corridor unreliability, wherein the plots exhibit the typical behavior of coherent systems. The life expectancy of the classical landscape network considered is found to be more than double that of a single corridor. Section 4 modifies the technique of Section 3 by allowing the paths to the new habitat patches to have a few edges in common. This is achieved by utilizing the Boole–Shannon expansion in the Boolean domain which resembles the total-probability theorem in the reliability domain. A discussion follows on the optimal allocation of reliability, which might be achieved either by constructing new corridors or by enhancing the reliability of the existing ones. The symbolic unreliability expressions obtained herein have been checked via the exhaustive tests set by Rushdi (1983b). Section 5 concludes the paper and points out new directions for further research.

2. Assumptions, notation, and nomenclature

2.1. Assumptions

- The analysis concerns one particular species, henceforth called the pertinent or concerned species. The analysis does not take into account any characteristic of the species.
- The pertinent species is in danger of local extinction in a certain habitat patch called the critical habitat patch. It escapes such extinction by migrating to a new habitat patch (one out of a few possible destination habitat patches) through imperfect corridors and perfect stepping stones.
- Each of the corridors is in one of two states, either good (permeable) or failed (deleted or destroyed).
- The migration system is also in one of two states, either successful or unsuccessful.
- Destination habitat patches and stepping stones are not susceptible to failure.
- Corridor states are statistically independent.

2.2. Notation

n = number of ecological corridors, $n \geq 1$.
X_i = success of corridor i = indicator that the concerned species successfully migrates through corridor i = a switching random variable that takes only one of the two discrete values 0 and 1 ($X_i = 1$ iff corridor i is permeable, while $X_i = 0$ iff corridor i is failed).
$\bar{X}_i$ = failure or deletion of corridor i = indicator variable for unsuccessful migration of the pertinent species through i, where $\bar{X}_i = 0$ iff corridor i is good, while $\bar{X}_i = 1$ iff corridor i is deleted/destroyed. The success X_i and the failure $\bar{X}_i$ are complementary variables.
$\boldsymbol{X}$ = a vector of n elements, each representing the successful species migration through a particular corridor i, $\boldsymbol{X} = [X_1\ X_2\ \ldots\ X_n]^T$.
$S(\boldsymbol{X})$ = indicator variable for the successful operation of the migration system (successful migration of the pertinent species), called system success.
$Pr[\ldots]$ = probability of the event $[\ldots]$.
$E[\ldots]$ = expectation of the random variable $[\ldots]$.
q_i, p_i = reliability and unreliability of corridor i; Both q_i and p_i are real values in the closed real interval [0.0, 1.0]. Here, we deliberately follow the variable definition of Jordán (2000) though