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Butterfly catastrophe model for wheat aphid population dynamics: Construction, analysis and application

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ABSTRACT

In agriculture, a population catastrophic phenomenon exists in many species of insects such as wheat aphids. Numerous previous attempts have been made to describe this dynamic behavior of insect populations using mathematical models in order to develop efficient biological control measures. However, most of the models are limited to no more than three controlling variables and restricted in theoretical analysis, thus, are not enough to cope with complicated ecological systems. Catastrophe theory, one of the earliest dynamic theories, can be used to address this problem more comprehensively. In this study, we propose using butterfly catastrophe theory to build a wheat aphid population dynamics model as a function of four controlling factors (natural enemy, weather factor, pesticide effect and carrying capacity). We used data collected by Ecology and Integrated Pest Management Laboratory in Northwest A & F University to verify the model. Model development, parameter estimation and verification results are presented. The results indicate that the butterfly catastrophe model can be applied to analyze aphid population dynamics considering four controlling variables. Effective management strategies for preventing catastrophic increase of wheat aphids can be carried out by changing the four controlling variables.

1. Introduction

As is well known, aphids are serious pests of agricultural crops, particularly in cereals including wheat (Emilio and Maria, 2014; Owain et al., 2008; Anne et al., 2010). Biological control strategies are essential to control them, ensuring an eco-friendly sustainable agricultural development. Populations of aphids are affected by many factors such as weather conditions, natural enemies, crop and intrinsic biological features (Nasir and Ahmad, 2001; Aheer et al., 2007, 2008; Ashfaq et al., 2007). Although these elements change frequently, the population of aphids does not always vary continuously. Sometimes a sudden outbreak may occur and sometimes the population may unexpectedly decline (Zhao et al., 1993, 2005; Zhao and Wang, 1993). This is referred as a catastrophe or a sudden jump of an aphid population (Wei et al., 2009). However, the existence of catastrophic behavior in a system does not mean that a catastrophe model is necessarily applicable unless at least one of two catastrophe flags exists (Glimore, 1981), one is called modality, and the other is hysteresis. Modality means that a system has multiple domains of attraction or multiple stable states (Glimore, 1981; May, 1977). Hysteresis occurs when a physical process is not strictly reversible (Glimore, 1981; Loehle, 1989). For instance, aphid population may increase suddenly with changing of controlling factors, but the population does not immediately return to the original level even if the control factors immediately return to the previous level.

Catastrophe theory originated as a branch of topology designed to deal with discontinuous dynamic systems governed by a potential function and can be used as a modeling approach to analyze complex nonlinear systems (Loehle, 1989; Arnold, 1984; Glimore, 1981). This theory has been applied in many fields including physics, biology, ecology, economics and medicine (Stewart, 1982; Deakin, 1980; Rosser, 2007; Jones, 1977). Although applications to study population dynamics in insect ecology, especially for aphids, are less documented, some significant examples can be found in Zhao et al. (1993, 2005), Zhao (1989, 1991), Zhao and Wang (1993), Ma and Bechinski (2009), Wei et al. (2009), and Piyaratne et al. (2013). Most of them concentrated on using the fold or the cusp theory with two control variables such as weather and natural enemy. Ma and Bechinski (2009) built a cusp catastrophe model to investigate the relationship between the intrinsic rate of increase

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of aphids *Diuraphis noxia* and environmental factors. They showed that population growth of aphids is intrinsically catastrophic with smooth change of temperature and plant growth stage. Zhao (1991), Zhao et al. (1993, 2005) and Zhao and Wang (1993) had studied the cusp model and provided a scientific basis for predicting and controlling pest insects by analyzing their dynamic protection threshold. On the other hand, applications of high-dimension models such as the swallowtail and the butterfly in aphid population dynamics are hardly found. The name "swallowtail" or "butterfly" means geometric shape of bifurcation sets in these models look like swallowtail or butterfly. The first butterfly catastrophe model had been constructed by Loehle (1989). Piyaratne et al. (2013) proposed a swallowtail model which was implemented as a computer software application (APHIDSim) considering three controlling factors. APHIDSim had been verified using actual field aphid population data analyzing the swallowtail behavior of aphid populations. Another attempt to construct a butterfly catastrophe model based on four controlling variables (natural enemy, weather factor, pesticide effect and crop condition) to analyze the dynamics of insect pest population had been reported in 2012 (Li et al., 2012). But that work had not focused on the estimation of model parameters. However, there are some issues in the validity and usefulness of catastrophe theory applications in ecological systems, and some of them had been explored by Loehle (1989) and Roopnarine (2008). Thus, it is clear that to have a more comprehensive understanding of aphid population change, it is necessary to combine practical data with the theoretical analysis. In this study, we developed a butterfly catastrophe model for the aphid population as a function of four controlling factors: natural enemy, weather factor, pesticide effect and carrying capacity (crop condition). The model has been verified using actual field data. Here we present a comprehensive analysis of aphid population dynamics in terms of the butterfly catastrophe theory including the model development and parameter estimation process and the verification results.

2. Materials and methods

2.1. Processes, variables and constants

The whole work presented in this paper consists of three sub models: the population dynamic model, the parameter estimation model and the butterfly catastrophe model. The population dynamic model is an improved version of the logistic equation (Eq. (1)) and it will be used to derive variables which are to be fitted in the butterfly catastrophe model. The parameter estimation model estimates the unknown parameters which are to be applied in the butterfly catastrophe model. Then we use the butterfly catastrophe model to analyze the dynamic behavior of aphid population change. The population equation is fitted to the equilibrium surface equation as a process between above sub models to analyze the population behavior of the butterfly catastrophe model.

2.2. The population dynamic model

In ecology, the well known logistic model (Verhulst, 1838 cited in Brauer and Castillo-Chavez, 2010) is widely used to describe population growth of insects because this model has a simple form with definite biological parameters and it can effectively reveal the relationship between population density and the change of population. Although the logistic equation has been applied in a wide range of ecological situations, its theoretical assumptions are relatively straightforward and open to many criticisms. Based on the basic model, varieties of modified models have been proposed by various authors (Cui et al., 1982; Smith, 1963). We proposed a new improved logistic model incorporating four influencing factors such as weather condition, crop condition (carrying capacity), predation (natural enemy) and pesticide effect (Eq. (1)) based on previous work done by Cui et al. (1982), Zhao et al. (2005) and Li et al. (2012).

$$(1)\,\frac{dN}{dt} = r(e)N\left(1-\frac{N^3}{K^3}\right) - \frac{Pq(N-N_m)}{(N-N_m)+d} - MN^2,$$

In this equation, N is the pest population density, $r(e)$ is the intrinsic growth rate which is a function of weather variable, K is the carrying capacity and M represents the pesticide factor. P, e and q are the predator population density, temperature and the rate of predation, respectively. N_m is the minimum pest population where the predation is possible, which is a constant. The half saturation point of predators, d, is also a constant.

According to the published literature, the individual increase rate, $(1/N)(dN/dt)$ is presented as a non-linear growth rate, $1-(N/K)^s\,(s>1)$, instead of a linear growth rate, $(1-(N/K))$ (Li et al., 1997; Zhao, 1989). Here we considered the non-linear growth rate where $s=3$ since it could easily fit the logistic model to the butterfly catastrophe model (see Section 2.3.1). The intrinsic growth rate, $r(e)$, is derived as a function of temperature (a detailed analysis is given in Section 2.4.2) since temperature is one of the most significant weather factors affecting the intrinsic growth rate of aphids (Yang and Ding, 1990). Having all these considerations, we introduced $r(e)N\left(1-\left(N^3/K^3\right)\right)$ as the key component to the change of population growth rate. According to Zhao et al. (2005), $Pq(N-N_m)/((N-N_m)+d)$ represents the predation of natural enemies. The impact of pesticides depicted as MN^2, is also considered.

2.3. Butterfly catastrophe model

We constructed the butterfly model from the aphid population as a function of four controlling factors including weather, carrying capacity, predation and pesticide effect, integrating the improved logistic growth equation (Eq. (1)) in order to analyze butterfly catastrophe behavior of aphid population dynamics. Based on butterfly theory, critical points dominating catastrophic behaviors exist in catastrophic regions where the first derivative of the potential function equals zero. The potential function of the butterfly catastrophe model (Saunders, 1980) is given by,

$$V(x:t,u,v,w) = x^6 + tx^4 + ux^3 + vx^2 + wx \tag{2}$$

where x is the state variable and t, u, v and w are the control variables. The equilibrium surface M is given by the first derivative, $V'(x:t,u,v,w)=0$, of the potential function of the butterfly model,

$$V'(x:t,u,v,w) = 6x^5 + 4tx^3 + 3ux^2 + 2vx + w = 0. \tag{3}$$

The singularity set S of all singular points is the subset of M which includes all the null points of the second derivative of the potential function. The singularity set of singular points is given by

$$S = \left\{x | V''(x) = 30x^4 + 12tx^2 + 6ux + 2v = 0\right\}. \tag{4}$$

The catastrophic behavior of aphid population is analyzed by mapping the four controlling factors (natural enemy, weather factor, pesticide effect and carrying capacity) space of the population dynamic model to the control variable space (t, u, v and w) in the bifurcation set of the butterfly model. The bifurcation set is obtained by eliminating x from Eqs. (3) and (4). Since the butterfly model has four control parameters, the bifurcation set is four-dimensional. The most effective approach to plot the bifurcation set is to exhibit intersection of B with a plane $u = c$ (c is a constant). In fact, when $u=0$, the intersection is able to reveal the main geometric characteristics of the butterfly model (Li et al., 2012). Let $u=0$, then bifurcation set B satisfies the equation

$$t\left(13824t^4 - 86400t^2v + 144000v^2\right)w^2 + 84375w^4$$
$$= v^3\left(-4096t^4 + 24567t^2v - 36864v^2\right). \tag{5}$$