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A model for coupling fire and insect outbreak in forests

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ABSTRACT

Predictive models of insect outbreak exist for some processes for few species, but an additional and rarely explored complication is the potential interaction between insect outbreak and wildfire disturbances in forests. The association between insect and fire dynamics is complex, particularly when evaluated over time and at large scale, and no consensus exists in the published literature about its consequences. Thus, more insights on the issue would be useful to scientists, resource managers, and the public when making decisions on, to name a few, firefighting, operations and treatments to reduce wildfire/insect impacts. In this article we propose mathematical models incorporating the effect of insect outbreaks either as a single disturbance in the forest population dynamics or coupled with wildfire disturbances. For the beetle-tree system model analytical and numerical characterization of its temporal dynamics shows that the system exhibits the well known beetle-tree interaction described by the dual equilibrium theory. For the extended model that includes fire, numerical simulations demonstrate the potential for existence of positive feedback between wildfire and insect outbreak disturbances in certain region of fire strength. This result agrees with one of the current theories in the field.

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1. Introduction

Forest disturbances have been recognized as a key factor affecting terrestrial biogeochemical processes. The issue on how disturbances alter terrestrial carbon cycles has recently attracted great attention from the research community because the global carbon cycle can be potentially destabilized by various disturbances (Liu et al., 2011; Luo and Weng, 2011; Weng et al., 2012). Most of the many studies that have been conducted to address this issue are largely at the descriptive and phenomenological levels. For example, most observational studies have focused on quantifying impacts of individual disturbance events on ecosystem carbon processes. So far, there is no mathematical framework to deal with positive and negative feedback among disturbances as well as interactions between climate and disturbances. Modeling these interactions is a challenging task, but an important one if we aim to quantitatively understand their impact on terrestrial biogeochemical processes (Liu et al., 2011; Luo and Weng, 2011).

A disturbance is defined as "any relatively discrete event in time that disrupts ecosystem, community, or population structure and changes resources, substrate availability, or the physical environment" (Liu et al., 2011). Disturbances are classified into natural (wildfire, windstorms, insect and diseases epidemics, floods, droughts, etc) and anthropogenic (deforestation, timber harvesting, application of herbicides, pollution, etc) disturbances.

Wildfire and insect infestations are two major natural disturbances of forest lands in the United States. Historically, insect infestations and wildfire have had a dominant influence on successional processes in forests of the Western United States. Fire suppression over the past 100 years has resulted in larger, more severe wildfire and insect outbreaks. In 2005, over 0.14 million hectares (0.34 million ac) of Federal lands in Oregon and Washington were affected by wildfire and approximately 0.8 million hectares (2.1 million acres) sustained damage from insects such as bark beetles and defoliators (Preisler et al., 2010). The mountain pine tree beetle, a native insect of the pine forest of western North America, has recently affected 130,000 km^2 in British Columbia. The spruce beetle and *Ips spp.*, bark beetle natives of western North America, have caused also extensive spatial damage. Another bark beetle that has caused forest damage over large regions in southern United States is the southern pine beetle (Liu et al., 2011).

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A reciprocal and synergistic association has frequently been described between fire and insects outbreak (Hicke et al., 2012; Preisler et al., 2010), but have been rarely explored in a mathematical modeling context (Liu et al., 2011). Despite potential feedbacks among these two natural disturbances, there is a lack of consensus in the published literature about responses, with some publications reporting large effects of beetle-killed trees on fuels and fire and other studies reporting no effect or a reduced impact (Hicke et al., 2012; Preisler et al., 2010). These results confirm that the association between insects and fire is a complex one, particularly when evaluated over time and at a large scale. Thus, more insights on the matter would be useful to scientists, resource managers, and the public when taking decisions on firefighting operations and treatments to reduce wildfire/insect outbreaks impacts. This fact motivated us to write this paper, which is a first step in addressing the general question: how do we incorporate into a mathematical framework positive and negative feedbacks among disturbances?

In this work we aim to lay down the foundations for investigating the effect of interaction between fires and mountain pine tree beetle outbreaks on the dynamics of the forests. In particular, we develop a mathematical model at the population level, to describe the effect of pine tree beetle on pine tree populations. This model has temporal dynamics that agrees with hypothetical two-phase model for beetle population dynamics (also known as the dual theory) (Berryman, 1979; MacQuarrie and Cooke, 2011; Raffa and Berryman, 1986) and fits the data available. In addition, the model includes ideas from a classic model introduced by Ludwig et al. (1978) and the model developed by Lynch (2006). The end model is simple enough to allow an analytical dynamical behavior characterization. Also, we extend the tree-beetle system model by incorporating "average" wildfire disturbances and their interaction with insect outbreak dynamics. The fire will kill both beetles and trees and will also weaken the defenses of the surviving trees to beetle infestations. Our numerical simulations of the extended model show that the presence of fire, depending on the strength of the fire rate, can change or not the dynamics of the tree-beetle system.

This paper is organized as follows. In Section 2.1 we introduce a mathematical model that describes the pine-tree beetle/pine-tree interaction, which constitutes the foundation for the extended model. In Section 2.2 we formulate the beetle-tree-fire extended model, where the wildfire is modeled as an "average" disturbance. In Section 3.1.1 we parameterize the beetle-tree model. Its temporal dynamical behavior characterization is given in Section 3.1.2 and is performed using standard dynamical system tools as well as numerical simulations. The main results concern the number of steady-state solutions (Result 3.3) and their stability (Result 3.4). The tool used to study the dynamical behavior of the beetle-tree-fire system is numerical simulations, which are presented in Section 3.1.3. We provide results discussion and future direction in Section 4.

2. Models description

We describe the formulation of the two deterministic models introduced in this paper: the model for tree-beetle system and the model for the tree-beetle-fire system. The former models the dynamics of pine-tree beetle/pine-tree interaction and the latter models the beetle-fire coupled disturbances on forests.

2.1. Deterministic model for tree-beetle system

In this section, we consider the effect of pine tree beetle on pine tree populations. We describe in detail the mathematical model and give a brief description of the model rescaling procedure.

2.1.1. Mathematical model

The equations of the model are

$$\frac{dV}{dt} = r_v V \left(1 - \frac{V}{K_v} - m(B)\right), \tag{1a}$$

$$\frac{dB}{dt} = r_b B \left(1 - \frac{B}{K_e}\right) - \frac{\alpha B^2}{1 + \beta B^2}. \tag{1b}$$

The state variables are V and B, denoting number of susceptible trees and mountain pine beetles per tree, respectively. Note that, at each time t, the total number of the beetles in the forest is given by $B_T(t) = B(t)V(t)$.

Eqs. (1a) and (1b) represent the dynamics of pine trees and beetles per tree, respectively. To construct the model of the time evolution of the tree-beetle system we used the number of beetles per tree data of Raffa and Berryman (1986) and of Mawby et al. (1989), we adapted the tree differential equation from Beckage et al. (2011), and incorporated the model ideas given in (Lynch, 2006, Section 4.2). In the tree growth Eq. (1a), the constant r_v is the intrinsic growth rate of susceptible trees (1/time) and K_v represents the carry capacity of the system. In absence of beetles the growth of the trees is density limited as given by a logistic self-interaction. The presence of beetles will introduce an additional mortality of trees given by the last term in (1a), which is assumed to be a function of the number of beetles through the saturation relation (Lynch, 2006):

$$m(B) = f_k \frac{B}{r + B}, \tag{2}$$

where f_k is the fraction of successfully attacked trees that are killed and r denotes the threshold of number of beetles for successful attack (beetles per tree). This specific relation models the fact that the additional mortality of trees due to the presence of beetles is linearly dependent on B for small number of beetles and it saturates at f_k when the number of beetles is very large. Substitution of (2) into ((1)) yields

$$\frac{dV}{dt} = r_v V \left[1 - \frac{V}{K_v} - \frac{f_k B}{r + B}\right], \tag{3a}$$

$$\frac{dB}{dt} = r_b B \left(1 - \frac{B}{K_e}\right) - \frac{\alpha B^2}{1 + \beta B^2}. \tag{3b}$$

The dynamics of the beetle population are modeled by (3b), which has the following attributes:

1. It constitutes one of the classic models for insect infestation in forests first developed by Ludwig et al. (1978). In the absence of any tree defense (tree defensive rate $\alpha = 0$), the growth of the beetles is density limited and modeled by a logistic equation. The tree defense modeled by the last term in (3b) depends on the number of beetles per tree and saturates for high numbers.
2. The differential equation provides a relationship between beetle numbers and changes in population density that agrees with the hypothetical two-phase model, also known as dual equilibrium theory, which describes the endemic-to-epidemic transition for the mountain pine beetle, *D. Ponderosae Hopkins*, bark beetles and other eruptive herbivores (Berryman, 1979; MacQuarrie and Cooke, 2011; Raffa and Berryman, 1986). According to this theory, population growth rates follow the standard nonlinear curve depicted in Fig. 1. That is, the growth is positive below a threshold K_T, which constitute a stable equilibrium. An increasing of beetle population beyond this level leads to a negative growth rate until the population density reaches K_1, which is an unstable steady-state. After the level K_1 the change in beetle population is again positive until the carrying capacity K_e is reached. K_e represents the stable epidemic equilibrium and after the beetle population