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Connectivity and stretch factor trade-offs in wireless sensor networks with directional antennae ☆

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ABSTRACT

We consider the following Antenna Orientation Problem: Given a connected Unit Disk Graph (UDG) formed by n identical omnidirectional sensors, what is the optimal range (or radius) which is necessary and sufficient for a given antenna beamwidth (or angle) ϕ so that after replacing the omnidirectional sensors by directional antennas of beamwidth ϕ it is possible to find an appropriate orientation of each antenna so that the resulting graph is strongly connected?

In this paper we study beamwidth/range tradeoffs for the Antenna Orientation Problem. Namely, for the full range of angles in the interval $[0, 2\pi]$ we compare the antenna range provided by an orientation algorithm to the optimal possible for the given beamwidth. We propose new antenna orientation algorithms that ensure improved bounds for given angle ranges and analyze their complexity.

We also examine the Antenna Orientation Problem with Constant Stretch Factor, where we wish to optimize both the transmission range and the hop-stretch factor of the induced communication network. We present approximations to this problem for antennas with angles $\pi/2 \le \phi \le 2\pi$.

1. Introduction

Traditional studies of wireless sensor networks (WSNs) have relied on the assumption that sensors transmit and receive using omnidirectional antennas. This leads to a communication model where a sensor s will be able to transmit information to every sensor situated within a circle centered at s with radius equal to its transmission range. Typically it is assumed that all sensors in a WSN have the same transmission range which results in an undirected communication graph that is a unit disk graph (UDG), where the unit is the transmission range, r.

In an attempt to improve overall network power consumption Caragiannis et al. [6] proposed replacing omnidirectional with directional antennas. Motivation for using directional antennas also comes from several recent studies; these include improvements in network capacity [16], reduction in neighbor interference [3], and enhancing the overall security [17] of the wireless network.

☆ This is an extended and synthesized version of results which originally appeared in [22] and [23].

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Directional antennas are modeled as circular sectors with an angle (beamwidth) ϕ. Whereas the underlying communication graph of an omnidirectional WSN is undirected, if transmission is carried out with directional antennas (reception remains omnidirectional), a directed communication graph emerges. Despite the fact that directional antennas can transmit farther than omnidirectional antennas using the same power, it is apparent that the resulting directed communication graph will not be connected unless the antennas are properly oriented. This raises the problem of devising antenna orientation algorithms which minimize the required transmission range and ensure a strongly connected communication network.

Despite the apparent simplicity of the Antenna Orientation Problem, relatively little is known on the optimal transmission range required for a given angle so as to attain the overall strong connectivity of the network. Caragiannis et al. [6] were the first to propose this problem and showed that it can be solved optimally for $\phi \geq 8\pi/5$, and is NP-Hard for $\phi < 2\pi/3$, where there is no approximation algorithm with ratio less than $\sqrt{3}$, unless $P = NP$. When $\phi = 0$ the antenna orientation problem is equivalent to the well-known Bottleneck Traveling Salesman Problem [15], where the an optimal approximation algorithm, given by Parker and Rardin [27], has an approximation ratio of 2. Several aspects of the Antenna Orientation Problem are addressed in the general survey [21]. An important generalization is studied in [11] when each sensor can be equipped with a given number of directional antennas which can be independently oriented. Tradeoffs between beamwidth, number of antennas, and minimum range required are considered.

Solutions to AOP guarantee strong connectivity for a WSN with directional antennas, but nothing more. We therefore also examine the Antenna Orientation Problem with Constant Stretch Factor – a problem similar to the Antenna Orientation Problem, but which also bounds the hop-stretch factor of the directional case compared to the omnidirectional case.

1.1. Preliminaries and notation

A *sensor* is an object at a point in the plane. It is able to receive transmissions omnidirectionally. A sensor with an omnidirectional antenna is able to transmit in all directions up to a distance r called the *transmission range*. A sensor with a single directional antenna is able to transmit in a sector whose angle is referred to as the *beamwidth* of the antenna. This antenna may be initially facing any direction, but once *oriented* the antenna is fixed in this orientation. Any point that lies within the sector defined by the antenna, regardless of its distance from the sensor, is in the sensor's *line of sight*.

For the purposes of this paper, any sensors referred to are assumed to be sensors with a single directional antenna. Furthermore, the term *sensor* may be used interchangeably to mean the location of the sensor in the plane, the sensor object itself, or the vertex representing the sensor in a graph. We may also use the terminology "orienting a sensor" to mean orienting the antenna at a sensor.

We assume that any sensor, s, is able to determine its distance from other sensors, as well as the angle formed by any other two sensors with vertex s. While assuming that sensors have location awareness will satisfy these assumptions, it is not strictly necessary as sensors do not need access to a global co-ordinate system for our results. Furthermore, we assume that each sensor has the ability to communicate with all nearby sensors during the orientation process. This could be accomplished through the rotation of its directional antenna, or the use of its omnidirectional antenna to transmit as well as receive.

Systems of omnidirectional antennas are modeled by using unit disk graphs (UDGs). We will use $UDG(P, r)$ to denote the UDG on the point-set P with unit r. We will be often referring to the neighborhood of a vertex v in a UDG. Since we do not know the arrangement of vertices beforehand, we will consider instead the area in which neighbors may be located – the closed disk of radius r centered at v, which we will denote $D[v, r]$.

Let a, b be sensors with range r. Sensor a *covers* sensor b if $b \in D[a, r]$ and b is within the line of sight of sensor a. This means that b will be a neighbor of a in the directed communication graph (although the reverse is not necessarily true). Sensor a covers area A if $\forall$ points $p \in A$, a sensor at p would be covered by a. A set of sensors S covers an area A if $\forall$ points $p \in A$, a sensor at p would be covered by at least one sensor $s \in S$.

Given a set of sensors S with beamwidth ϕ, let $\mathcal{O}$ be an orientation of every sensor in S. We denote the communication graph induced by $\mathcal{O}$ on S as $G_{\mathcal{O}}(S, E_{\mathcal{O}})$, where a directed edge $\overrightarrow{(u, v)} \in E_{\mathcal{O}}$ if and only if u covers v in the orientation $\mathcal{O}$.

We say that a can *reach* b if a covers b, or if a covers a sensor c and c can reach b. In general terms, this means that a can reach b if there exists a directed path from a to b in the directed communication graph.

We now formally define the Antenna Orientation Problem.

Antenna Orientation Problem. Let S be a set of connected sensors in the plane with its communication network represented by the unit disk graph $U(S)$. Given ϕ, find the minimum transmission range, denoted by $r(U(S), \phi)$, which is necessary and sufficient so that after replacing the omnidirectional transmission antennas of S by directional antennas of beamwidth ϕ there exists an appropriate orientation of each sensor, denoted by $\mathcal{O}$, so that the resulting communication network $G_{\mathcal{O}}(S, E_{\mathcal{O}})$ is strongly connected.

Before defining the Antenna Orientation Problem with Constant Stretch Factor, we must first define what we mean by stretch factor.

For P, a set of points, let $U(P)$ be the UDG on P where the unit is the longest edge of the MST of P. Let $G(P)$ be a (strongly) connected (di)graph on the vertices in P. For any two vertices u, v in P, let $d_G(u, v)$ denote the minimum