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Short communication

Improvements in anisotropic models of single tree effects in Cartesian coordinates

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ABSTRACT

For anisotropic density functions of e.g. fruit or leaf dispersal, most mathematical research is only done in polar coordinates. However, in software solutions aiming to derive inverse models for real world dispersal data, Cartesian coordinates may be preferred for several reasons. Thus, we introduce an anisotropic model in Cartesian coordinates following the approach in Wälder et al. (2009) with the von Mises approach. By introducing a correction factor, we thereby consider the fundamental attribute, that the integral over a density function with respect to the Cartesian coordinates has to be equal 1. It may have been overlooked so far that guaranteeing for this attribute needs different approaches whether working in polar or Cartesian coordinates. One result is that our approach can be used also for other anisotropic models rather than models from the von Mises approach.

1. Introduction

Individually based models have reached widespread acceptance so far (e.g. in simulations). However, to improve accuracy of predictions, the underlying individual effect models need to have highest ecological consistency available. An important step to improve so called inverse models (Clark et al., 1999) was done by widening applicability through incorporating anisotropic algorithms instead of solely using isotropic ones (Wagner et al., 2004).

When inverse models are applied to real-world data (e.g. in seed or leaf dispersal), the scientist often wants to deduce the total amount of subjects an individual produces. To do so, some prerequisites have to be met by the kernel formula. One of those prerequisites is that the integral over the density function with respect to the Cartesian coordinates has to equal 1.

Although this aspect seems clear, the proof whether the models fit this demand is hardly ever given by the authors.

2. Theory

Based on the anisotropic model for fruit dispersal in polar coordinates of Wälder et al. (2009), we want to deduce an anisotropic model in Cartesian coordinates. To get a better understanding of our approach, we give a brief description of the work done in Wälder et al. (2009).

They started with the established density function in Cartesian coordinates

$$p_i(x,y) := \frac{\exp\left(-\left(\left(\ln(\sqrt{x^2+y^2})-\mu^2\right)/2\sigma^2\right)\right)}{(2\Pi)^{3/2}\sigma(x^2+y^2)} \tag{1}$$

for the lognormal model. This function is equivalent by coordinate transformation to

$$p_i(r) := \frac{\exp\left(-\left((\ln(r)-\mu)^2/2\sigma^2\right)\right)}{(2\Pi)^{3/2}\sigma r^2} \tag{2}$$

which is the density function in polar coordinates mentioned in Stoyan and Wagner (2001: 38). Then they stated the important attribute of the density function. The integral over the density function with respect to the Cartesian coordinates has to be equal 1,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_i(x,y)\,dy\,dx = \int_0^{2\pi}\int_0^{\infty} p_i(r)\cdot r\,dr\,d\varphi = 1. \tag{3}$$

The equality in (3) holds because of the Jacobian of $x = r\cdot\cos\varphi$ and $y = r\cdot\sin\varphi$

$$\det J = \det\frac{\partial(x,y)}{\partial(r,\varphi)} = \begin{vmatrix}\cos\varphi & -r\sin\varphi\\ \sin\varphi & r\cos\varphi\end{vmatrix} = r. \tag{4}$$

After introducing the modified distance $r(\varphi) = r\cdot f(\varphi)$ with the *von Mises approach*

$$f(\varphi) = \exp(k\cos(\varphi - u + \pi)), \tag{5}$$

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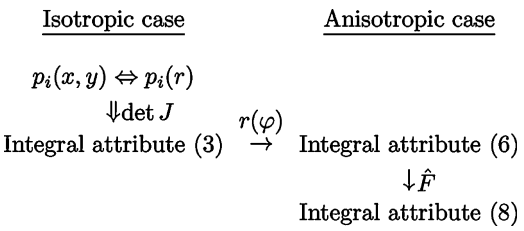

Fig. 1. Procedure of the approach from Wälder et al. (2009); ⇔ equivalence, ⇓ mathematical relation, → modification.

they replaced r by the modified distance $r(\varphi)$ in the integral of Eq. (3) and got

$$\int_0^{2\pi}\int_0^{\infty} p_a(r(\varphi))\cdot r(\varphi)\,\mathrm{d}r\,\mathrm{d}\varphi \neq 1 \tag{6}$$

with

$$p_a(r(\varphi)) := \frac{\exp\left(-\left((\ln(r(\varphi))-\mu)^2\right)/2\sigma^2\right)}{(2\Pi)^{3/2}\sigma(r(\varphi))^2}. \tag{7}$$

To correct this problem they established the correction factor $\hat{F} = 2\pi/(\int_0^{2\pi} 1/f(\varphi)\,\mathrm{d}\varphi)$ and got

$$\int_0^{2\pi}\int_0^{\infty} \hat{F}\cdot p_a(r(\varphi))\cdot r(\varphi)\,\mathrm{d}r\,\mathrm{d}\varphi = 1. \tag{8}$$

The summary of this approach can be seen in Fig. 1.

Our approach is slightly different. We start from the density function $p_i(x, y)$ in (1) with the Cartesian coordinates (x, y) by assuming that the tree position is $(0, 0)$. To describe the anisotropic density function with the von Mises approach in (5) in Cartesian coordinates, we use the well known transformations

$$\varphi = \varphi(x,y) := \begin{cases} \arctan\left(y/x\right), & x>0;\\ \arctan\left(y/x\right)+\pi, & x<0;\\ \pi/2, & x=0, y>0;\\ 3\pi/2, & x=0, y\leq 0; \end{cases}$$

and $r = \sqrt{x^2+y^2}$. Thus, in function (1) we replace the distance $\sqrt{x^2+y^2}$ by the modified distance $d(x,y) := \sqrt{x^2+y^2}\cdot f(\varphi(x,y))$, which is equivalent to $r(\varphi)$, and get

$$p_a(x,y) := \frac{\exp\left(-\left(\left((\ln(d(x,y))-\mu)^2\right)/2\sigma^2\right)\right)}{(2\Pi)^{3/2}\sigma(d(x,y))^2}. \tag{9}$$

This anisotropic density function is equivalent to (7). The modification from the isotropic density function to the anisotropic density function can be seen by comparing Figs. 2 and 3. In both figures, the

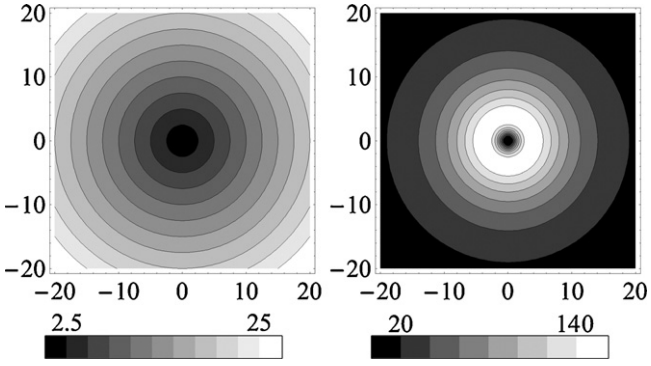

Fig. 2. Contourplots of the distance r in meter (left picture) and the seed distribution $M\cdot p_i(r)$ in numbers per square meter (right picture) with $M = 100{,}000$, $\mu = 2.6$ and $\sigma = 0.8$.

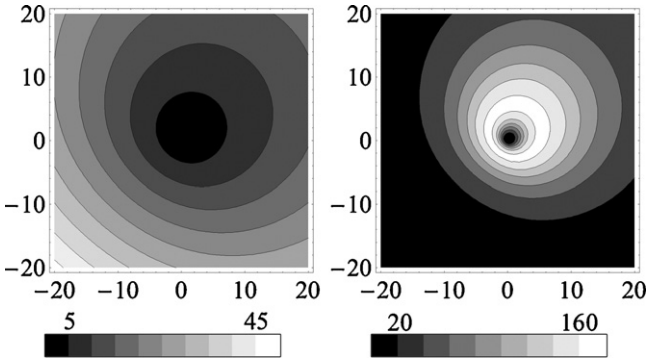

Fig. 3. Contourplots of the modified distance $r(\varphi)$ in meter (left picture) and the seed distribution $F\cdot M\cdot p_a(r(\varphi))$ in numbers per square meter (right picture) with $M = 100{,}000$, $\mu = 2.6$, $\sigma = 0.8$, $k = 0.5$ and $u = 0.9$.

distance function r and $r(\varphi)$ respectively is shown on the left side and the associated density distribution of the seeds, i.e. the density function multiplied by the total seed amount $M = 100{,}000$ of one tree $M\cdot p_i(r)$ and $M\cdot p_a(r(\varphi))$, is shown on the right side.

Checking the integral attribute for our density function (9) we got

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_a(x,y)\,\mathrm{d}y\,\mathrm{d}x \neq 1. \tag{10}$$

Thus we introduce the correction factor

$$F = \frac{1}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_a(x,y)\,\mathrm{d}y\,\mathrm{d}x}. \tag{11}$$

By considering $\det J$ we transform Eq. (11) in polar coordinates to get a closer form

$$F = \frac{1}{\int_0^{2\pi}\int_0^{\infty} p_a(r(\varphi))\cdot r\,\mathrm{d}r\,\mathrm{d}\varphi}.$$

This can be reduced to $F = 2\pi/(\int_0^{2\pi} 1/(f(\varphi))^2\,\mathrm{d}\varphi)$. Now the following holds

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F\cdot p_a(x,y)\,\mathrm{d}y\,\mathrm{d}x = \int_0^{2\pi}\int_0^{\infty} F\cdot p_a(r(\varphi))\cdot r\,\mathrm{d}r\,\mathrm{d}\varphi = 1. \tag{12}$$

We summarize the procedure of our approach in Fig. 4.

3. Results and conclusion

From our point of view we want to stress two important things. The first one is a comparison with the results in Wälder et al. (2009) while the other one alludes to our correction factor F.

First we want to stress out what influence the different correction factors $\hat{F}$ and F have. We consider the example of Fig. 3. Selecting the correction factor of Wälder et al. (2009) the amount

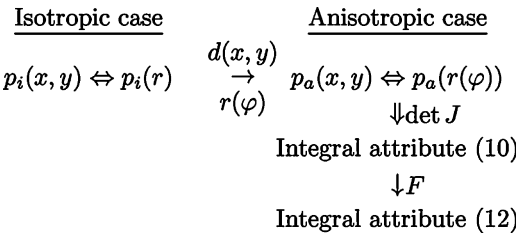

Fig. 4. Procedure of our approach; ⇔ equivalence, ⇓ mathematical relation, → modification.