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Every edge lies on cycles embedding in folded hypercubes with vertex-fault-tolerant

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The folded hypercube is a well-known variation of hypercube structure and can be constructed from a hypercube by adding a link to every pair of vertices with complementary addresses. An n-dimensional folded hypercube (FQ_n for short) for any odd n is known to be bipartite. In this paper, let f be a faulty vertex in FQ_n. It has been shown that (1) Every edge of $FQ_n - \{f\}$ lies on a fault-free cycle of every even length l with $4 \leq l \leq 2^n - 2$ where $n \geq 3$; (2) Every edge of $FQ_n - \{f\}$ lies on a fault-free cycle of every odd length l with $n + 1 \leq l \leq 2^n - 1$, where $n \geq 2$ is even. In terms of every edge lies on a fault-free cycle of every odd length in $FQ_n - \{f\}$, our result improves the result of Cheng et al. (2013) where odd cycle length up to $2^n - 3$.

1. Introduction

Design of *interconnection networks* (*networks* for short) is an important integral part of parallel processing and distributed systems. The *hypercube* is a well-known interconnection network model. The hypercube has several excellent properties, such as recursive structure, regularity, symmetry, small diameter, short mean internode distance, low degree, and much smaller edge complexity, which are very important for designing massively parallel or distributed systems [16]. Numerous variants of the hypercube have been proposed in the literature [3,4,19]. One variant that has been the focus of a great deal of research is the *folded hypercube*, which can be constructed from a hypercube by adding a link to every pair of nodes that are the farthest apart, i.e., two nodes with complementary addresses. The folded hypercube has been shown to be able to improve the system's performance over a regular hypercube in many measurements, such as diameter, fault diameter, connectivity, and so on [3,22].

An important feature of an interconnection network is its ability to efficiently simulate algorithms designed for other architectures. Such a simulation can be formulated as *network embedding*. An *embedding* of a *guest network* G into a *host network* H is defined as a one-to-one mapping f from nodes in G into nodes in H so that a link of G corresponds to a path of H under f [16]. The embedding strategy allows us to emulate the effect of a guest graph on a host graph. Then, algorithms developed for a guest graph can also be executed well on the host graph.

Linear arrays and rings, which are two of the most fundamental networks for parallel and distributed computation, are suitable for designing simple algorithms with low communication costs. Numerous efficient algorithms designed on linear arrays and rings for solving various algebraic problems and graph problems can be found in [16]. These algorithms can

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be used as control/data flow structures for distributed computing in arbitrary networks. An application of longest paths to a practical problem was encountered in the on-line optimization of a complex Flexible Manufacturing System [1]. These applications motivate the embedding of paths and cycles in networks.

Since faults may occur when a network is put into use, it is practically meaningful and important to consider faulty networks. Previously, the problem of fault-tolerant cycle embedding on an n-dimensional folded hypercube FQ_n has been studied in [7–9,11–15,22,25,26].

Let FF_v and FF_e be the sets of faulty vertices and faulty edges of FQ_n. Dajin Wang [22] showed that $FQ_n - FF_e$[1] contains a Hamiltonian cycle of length 2^n if $|FF_e| \le n-1$. Ma [18] showed that $FQ_n - FF_e$ contains a Hamiltonian cycle of length 2^n where each vertex is incident with at least two fault-free edges, when $|FF_e| \le 2n-3$. Hsieh [8] showed that $FQ_n - FF_e$ remains Hamiltonian-connected if $|FF_e| \le n-2$, where $n \ge 2$ is even, and showed that $FQ_n - FF_e$ remains strongly (respectively, hyper) Hamiltonian-laceable if $|FF_e| \le n-1$ (respectively, $|FF_e| \le n-2$), where $n \ge 3$ is odd. Fu [5] showed that $FQ_n - FF_e - FF_v$ contains a cycle of length at least $2^n - 2|FF_v|$ if $|FF_e| \le n-1$ and $|FF_v| + |FF_e| \le 2n-4$. Xu [24] showed that every edge of FQ_n lies on a cycle of every even length from 4 to 2^n; if n is even, every edge of FQ_n also lies on a cycle of every odd length from $n+1$ to 2^n-1. After that Xu [25] extended the above result to show that every fault-free edge of $FQ_n - FF_e$ lies on a cycle of every even length from 4 to 2^n; if n is even, every edge of $FQ_n - FF_e$ also lies on a cycle of every odd length from $n+1$ to 2^n-1, where $|FF_e| \le n-1$. Recently, Cheng [2] showed that every fault-free edge of $FQ_n - FF_v$ lies on a cycle of every even length from 4 to $2^n - 2|FF_v|$ if $n \ge 3$, and if $n \ge 2$ is even, every edge of $FQ_n - FF_v$ also lies on a cycle of every odd length from $n+1$ to $2^n - 2|FF_v| - 1$, where $|FF_v| \le n-2$. In this paper, we extend Cheng's [2] results to embedding more cycles on FQ_n with faulty vertex f. We obtain the following two properties:

1. Every edge of $FQ_n - \{f\}$ lies on a fault-free cycle of every even length l with $4 \le l \le 2^n - 2$ where $n \ge 3$;
2. Every edge of $FQ_n - \{f\}$ lies on a fault-free cycle of every odd length l with $n+1 \le l \le 2^n - 1$, where $n \ge 2$ is even.

Throughout this paper, a number of terms—network and graph, node and vertex, edge and link—are used interchangeably. The remainder of this paper is organized as follows: in Section 2, we provide some necessary definitions and notations. We present our main result in Section 3. Some concluding remarks are given in Section 4.

2. Preliminaries

A graph $G = (V, E)$ is an ordered pair in which V is a finite set and E is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. We say that V is the *vertex set* and E is the *edge set*. We also use $V(G)$ and $E(G)$ to denote the vertex set and edge set of G, respectively. Two vertices u and v are *adjacent* if $(u, v) \in E$. A graph $G = (V_0 \cup V_1, E)$ is bipartite if $V_0 \cap V_1 = \emptyset$ and $E \subseteq \{(x, y) \mid x \in V_0$ and $y \in V_1\}$. A path $P[v_0, v_k] = \langle v_0, v_1, \ldots, v_k \rangle$ is a sequence of distinct vertices in which any two consecutive vertices are adjacent. We call v_0 and v_k the *end-vertices* of the path. In addition, a path may contain a *subpath*, denoted as $\langle v_0, v_1, \ldots, v_i, P[v_i, v_j], v_j, v_{j+1}, \ldots, v_k \rangle$, where $P[v_i, v_j] = \langle v_i, v_{i+1}, \ldots, v_{j-1}, v_j \rangle$. The length of a path is the number of edges on the path. A path $\langle v_0, v_1, \ldots, v_k \rangle$ forms a *cycle* if $v_0 = v_k$ and $v_0, v_1, \ldots, v_{k-1}$ are distinct. A vertex is *fault-free* if it is not faulty. An edge is *fault-free* if the two end-vertices and the edge between them are not faulty. A path (respectively, cycle) is *fault-free* if it contains no faulty edges.

A bipartite graph G is *Hamiltonian-laceable* if there exists a Hamiltonian path between any two vertices from different partite sets. A Hamiltonian-laceable graph $G = (V_0 \cup V_1, E)$ is *strong* [6] if there is a simple path of length $|V_0| + |V_1| - 2$ between any two nodes of the same partite set. A Hamiltonian-laceable graph $G = (V_0 \cup V_1, E)$ is *hyper-Hamiltonian laceable* [17] if for any vertex $v \in V_i$, $i = 0, 1$, there is a Hamiltonian path of $G - v$[2] between any two vertices of V_{1-i}. For graph-theoretic terminologies and notations not mentioned here, see [23].

An *n-dimensional hypercube* Q_n can be represented as an undirected graph such that $V(Q_n)$ consists of 2^n vertices which are labeled as binary strings of length n from $\underbrace{00\ldots0}_{n}$ to $\underbrace{11\ldots1}_{n}$. Each edge $e = (u, v) \in E(Q_n)$ connects two vertices u and v if and only if u and v differ in exactly one bit of their labels, i.e., $u = b_n b_{n-1} \ldots b_k \ldots b_1$ and $v = b_n b_{n-1} \ldots \overline{b_k} \ldots b_1$, where $\overline{b_k}$ is the *one's complement* of b_k, i.e., $\overline{b_k} = 1 - i$ iff $b_k = i$ for $i = 0, 1$. We call that e is an edge of *dimension* k. Clearly, each vertex connects to exactly n other vertices. In addition, there are 2^{n-1} edges in each dimension and $|E(Q_n)| = n \cdot 2^{n-1}$. Fig. 1 shows a 2-dimensional hypercube Q_2 and a 3-dimensional hypercube Q_3.

Let $x = x_n x_{n-1} \ldots x_1$ be an n-bit binary string. For $1 \le k \le n$, we use $x^{(k)}$ (respectively, $\bar{x}$) to denote the binary strings $y_n y_{n-1} \ldots y_1$ such that $y_k = 1 - x_k$ and $x_i = y_i$ for all $i \ne k$ (respectively, $y_i = 1 - x_i$ for all $1 \le i \le n$). The *Hamming distance* $h(x, y)$ between two vertices x and y is the number of different bits in the corresponding strings of both vertices. The *Hamming weight* $hw(x)$ of x is the number of i's such that $x_i = 1$. Note that Q_n is a bipartite graph with two partite sets $\{x \mid hw(x)$ is odd$\}$ and $\{x \mid hw(x)$ is even$\}$. Let $d_{Q_n}(x, y)$ be the *distance* between two vertices x and y in graph Q_n. Clearly, $d_{Q_n}(x, y) = h(x, y)$.

[1] The graph obtained by deleting FF_e from FQ_n.
[2] The graph obtained by deleting v from G.