



Fan-planarity: Properties and complexity [☆]



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ABSTRACT

In a *fan-planar drawing* of a graph an edge can cross only edges with a common end-vertex. Fan-planar drawings have been recently introduced by Kaufmann and Ueckerdt [35], who proved that every n -vertex fan-planar drawing has at most $5n - 10$ edges, and that this bound is tight for $n \geq 20$. We extend their result from both the combinatorial and the algorithmic point of view. We prove tight bounds on the density of constrained versions of fan-planar drawings and study the relationship between fan-planarity and k -planarity. Also, we prove that testing fan-planarity in the variable embedding setting is NP-complete.

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1. Introduction

There is a growing interest in the study of non-planar drawings of graphs with forbidden crossing configurations. The idea is to relax the planarity constraint by allowing edge crossings that do not affect too much the drawing readability. Among the most popular types of non-planar drawings studied so far we recall:

- k -planar drawings, where an edge can have at most k crossings (see, e.g., [5,8,9,15,22,24,28,33,34,36,37,40]);
- k -quasi-planar drawings, which do not contain k mutually crossing edges (see, e.g., [1,3,4,21,30,41]);
- RAC (Right Angle Crossing) drawings, where edges can cross only at right angles (see, e.g., [25] and [26] for a survey);
- ACE_α drawings [2] and ACL_α drawings [6,20,27], which are generalizations of RAC drawings; namely, in an ACE_α drawing edges can cross only at an angle that is exactly α ($\alpha \in (0, \pi/2]$); in an ACL_α drawing edges can cross only at angles that are at least α (see also [26]);
- fan-crossing free drawings, where an edge cannot cross two other edges having a common end-vertex [16].

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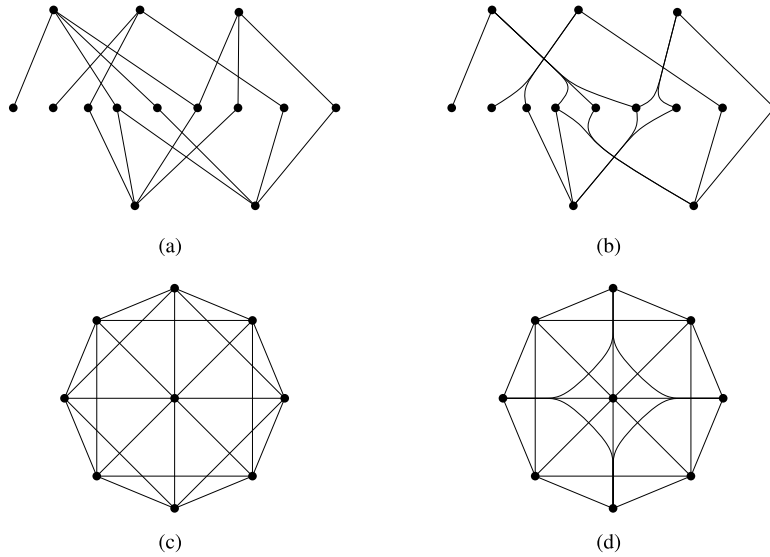


Fig. 1. (a) A fan-planar drawing of a graph G with 12 crossings. (b) A confluent drawing of G with 3 crossings. (c) A fan-planar drawing with 16 crossings of another graph G . (d) A confluent drawing of G with 8 crossings.

Given a desired type T of non-planar drawing with forbidden crossing configurations, a classical combinatorial problem is to establish bounds on the maximum number of edges that a drawing of type T can have; this problem is usually dubbed a Turán-type problem, and several tight bounds have been proved for the types of drawings mentioned above, both for edges drawn as straight-line segments and for edges drawn as polylines (see, e.g., [1,2,4,15,16,24,25,27,30,37,41]). From the algorithmic point of view, the complexity of testing whether a graph G admits a drawing of type T is one of the most interesting problem. Also for this problem several results have been shown, both in the *fixed* and in the *variable embedding setting* (see, e.g., [8,18,19,32,33,36]). In the fixed embedding setting, for each vertex v , the circular ordering of the edges incident to v is given as part of the input and cannot be changed; conversely, in the variable embedding setting such an ordering can be freely chosen.

In this paper we investigate *fan-planar drawings* of graphs, in which an edge cannot cross two independent edges, i.e., an edge can cross several edges provided that they have a common end-vertex. Fan-planar drawings have been recently introduced by Kaufmann and Ueckerdt [35]; they proved that every n -vertex graph without loops and multiple edges that admits a fan-planar drawing has at most $5n - 10$ edges, and that this bound is tight for $n \geq 20$. Fan-planar drawings are on the opposite side of fan-crossing free drawings mentioned above. Besides its intrinsic theoretical interest, we observe that fan-planarity can be also used in many cases for creating drawings with few edge crossings per edge in a confluent drawing style (see, e.g., [23,29]). For example, Fig. 1(a) shows a fan-planar drawing Γ with 12 crossings; Fig. 1(b) shows a new drawing with just 3 crossings obtained from Γ by bundling crossing “fans”. Another example is shown in Figs. 1(c) and 1(d).

We prove both combinatorial properties and complexity results related to fan-planar drawings of graphs. The main contributions of our work are as follows:

- We study the density of constrained versions of fan-planar drawings, namely *outer fan-planar drawings*, where all vertices must lie on the external boundary of the drawing, and *2-layer fan-planar drawings*, where vertices are placed on two distinct horizontal lines and edges are vertically monotone lines. We prove tight bounds for the edge density of these drawings. Namely, we show that n -vertex outer fan-planar drawings have at most $3n - 5$ edges (a tight bound for $n \geq 5$), and that n -vertex 2-layer fan-planar drawings have at most $2n - 4$ edges (a tight bound for $n \geq 3$). We remark that outer and 2-layer non-planar drawings have been previously studied in the 1-planarity setting [8,24,33] and in the RAC planarity setting [18,19].
- Since general fan-planar drawable graphs have at most $5n - 10$ edges and the same bound holds for 2-planar drawable graphs [37], we investigate the relationship between these two graph classes (observe that 1-planar graphs are always fan-planar by definition). More in general, we study the relationship between k -planarity and fan-planarity, proving that in fact for any $k \geq 2$ there exist fan-planar drawable graphs that are not k -planar, and vice versa.
- Finally, we show that testing whether a graph admits a fan-planar drawing in the variable embedding setting is NP-complete.

The rest of the paper is structured as follows: In Section 2 we give some preliminary definitions. Section 3 describes the tight bounds on the edge density of outer and 2-layer fan-planar drawable graphs. The relationship between k -planarity and fan-planarity is shown in Section 4, while Section 5 proves the NP-completeness of the fan-planarity testing problem.

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