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A discrete model for estimating the development time from egg to infecting larva of *Ostertagia ostertagi* parametrized using a fuzzy rule-based system

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ABSTRACT

Ostertagia ostertagi is a nematode, predominantly affecting cattle in the Pampean region of Argentina. A mathematical model parametrized using fuzzy rule-based systems of the Takagi-Sugeno-Kant type (FTSK) for estimating the development time from egg to infecting larval stage L3 of the gastrointestinal parasite *O. ostertagi* is here proposed. The estimation of development time of *O. ostertagi* is essential for the generation of appropriate control mechanisms, since this provides information about the time when parasites are ready to migrate to pastures. For the purpose of reflecting the natural environmental conditions, the mean daily temperature is taken as the main and only regulator of the development time. Humidity conditions are considered to be sufficient for the normal development of the larvae. Hence the individual's daily growth is a function of its length and the mean temperature recorded on the previous day. It is expressed in terms of a difference equation with fuzzy parameters, which are defined using laboratory data. Model outputs are tested against results of field experiments. Simulation results are very satisfactory, yielding a mean estimation error (MEE) of 0.64 weeks, with variance 0.34, and a determination coefficient $R^2 = 0.74$. The model clearly exhibits an inverse relationship between development time and temperature both in controlled and in field conditions. It also exhibits a very sensitive response both to the order in which the temperature sequence occurs, – reproducing the differences observed between spring and autumn – and to the amplitude of the temperature range.

1. Introduction

The gastrointestinal nematode parasite *Ostertagia ostertagi* is predominant in the Pampean region (Argentina) (Fiel et al., 1994). From the economic point of view, it is considered the most important nematode affecting cattle in temperate regions (Steffan et al., 1982; Entrocasso, 1981) and as such, much effort has been devoted to investigating this parasite species.

The cycle life of *O. ostertagi* is direct, without an intermediary host. There are two distinct stages: the *free-living stage* (*Egg-L1-L2-L3*) and the *parasitic stage* (*L4-L5-Adult*). The free-living stage occurs on the ground, first within the dung-pat and later on the grass. Larvae in the L1 and L2 stages feed on fungi and bacteria. The infective larva (L3) is ensheathed and does not feed. Following ingestion by cattle, L3 larvae undergo a process of exsheathment in the rumen before the fourth parasitic stage begins (*L4*). *Afterwards* larvae quickly develop into the adult stage.

The free-living stage has been studied under both controlled and field conditions (Fiel et al., 2008; Rossanigo and Gruner, 1995; Gibson, 1981; Young et al., 1980a, 1980b; Pandey, 1972; Rose, 1969). Several studies have revealed that there is a direct nonlinear relationship between development time and temperature over the range from 5 °C to 35 °C (Fiel et al., 2008; Williams, 1983; Catto, 1982; Pandey, 1972).

The estimation of development time is essential for the generation of appropriate control mechanisms, since this provides information about the time when parasites are ready to migrate to pastures. The variability of responses to different environmental factors makes the use of modelling tools relevant to help understand the complexity of the dynamics of the life cycle of the parasite. Development times can vary from 5 days for larvae under ideally warm and controlled conditions (Williams et al., 1987) to 36 days for larvae at a constant low temperature of 5 °C (Young et al., 1980b). Beyond these limits the mortality rate is high (Levine, 1978). The ideal temperature is within the range of 20–30 °C (Pandey, 1972; Rose, 1969) while the development

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process is difficult to accomplish at temperatures below 5 °C and above 40 °C (Pandey, 1972). Hence, during the warm months development takes only a few days while in winter the period increases to several weeks, especially if the winter is cold and wet (Catto, 1982; Durie, 1961).

In the literature, the models used for estimating development times are mostly of statistical type, strongly based on data from trials performed under controlled conditions, yielding a good fit to the data. Since these models strongly depend on the data, they are disadvantageous when it becomes necessary to extrapolate.

Here we propose a mathematical model, based on difference equations and a fuzzy rule-based system (FRBS), for estimating the development time from egg to L3 of the gastrointestinal parasite *Ostertagia* which adapts very adequately to the environmental conditions in the Pampean region of Argentina. The model consists of a difference equation and the parameters are functional forms defined through a FRBS which incorporates both quantitative and qualitative information on the processes involved. These fuzzy parameters allow for the flexibility needed when attempting to replicate field conditions, which are fundamental for developing any control strategy. The concept on which this model is based allows it to respond efficiently to wide temperature ranges.

2. Methods

2.1. Model description

The length of the larva is taken as an indicator of development, owing to the parasite's elongated shape. If the lengths at hatching and at the time of reaching the infective stage (L3) are known, then it is possible to determine how many days an individual larva needs to be able to complete its development under a particular environment (characterized by temperature). Larvae increase their mobility as the time of their development to L3 is nearly over and, therefore their development rate increases as they age.

If $L_t(a)$ is the length of a larva that was born on day t and is now aged a, then its growth is described by the difference equation:

$$L_t(a+1) = L_t(a) + r(T_{(t+a)})L_t(a) = \left\lfloor 1 + rT_{(t+a)} \right\rfloor L_t(a)$$
$$IC \quad L_t(0) = l_0(T_t)$$

where $r(T_{(t+a)})$ is the development rate, which depends on $T_{(t+a)}$ the average temperature of the current day $t+a$, and $l_0(T_t)$ is the hatching length. The difference equation is solved using a daily step. Clearly all the larvae in same cohort (born the same day) grow exactly at the same pace.

This equation has two border conditions: an initial condition, $l_0(T_t)$, and a final condition, $l_{L3}(T)$, the length of a larva when reaching the L3 stage. It is worth mentioning that these conditions also vary depending on the environmental temperature.

Once a day t is fixed, the age a at which the larva reaches the L3 stage is such that $L_t(a) < l_{L3}(T) \leq L_t(a+1)$. Hence, if $\tau(t)$ is the development time of a larva which was born on day t, then $\tau(t) = a+1$.

The model was implemented using GNU Scilab 4.1. The inputs of the model are the daily average temperatures, which are loaded as a vector. The program runs the simulation with two output options:

1. Creates a graph which shows the time of development for each cohort hatching each day within the range of the temperature vector.
2. Creates a graph which describes the development of one specific cohort starting on a selected Julian day; the temperature vector must start on that day.

The temperature vector should be long enough so that the full development of the larvae can be attained.

Table 1
Definition of all membership functions used in the parameterization of the model. Function types are as defined in Appendix B.

Name of membership function	Type	Parameter values		
		a	*b*	*c*
Temp10	$z(x;a,b)$	10	15	
Temp15	$T(x;a,b,c)$	10	15	20
Temp20	$T(x;a,b,c)$	15	20	25
Temp25	$T(x;a,b,c)$	20	25	30
Temp30	$T(x;a,b,c)$	25	30	35
Temp35	$s(x;a,b)$	30	35	
$f_{[10]}$	$L(x;a,b)$	6	323	
$f_{[15]}$	$L(x;a,b)$	−13.8	620	
$f_{[20]}$	$L(x;a,b)$	1.2	320	
$f_{[25]}$	$L(x;a,b)$	−4.6	465	
$f_{[30]}$	$L(x;a,b)$	−2.4	399	
$f_{[35]}$	$L(x;a,b)$	9		
$g_{[10]}$	$L(x;a,b)$	4	799	
$g_{[15]}$	$L(x;a,b)$	2.8	817	
$g_{[20]}$	$L(x;a,b)$	−5.8	989	
$g_{[25]}$	$L(x;a,b)$	−3.6	934	
$g_{[30]}$	$L(x;a,b)$	−13.8	1240	
$g_{[35]}$	$L(x;a,b)$	21.62		
$h_{[10]}$	$L(x;a,b)$	0.0036		
$h_{[15]}$	$L(x;a,b)$	0.0184	−0.2	
$h_{[20]}$	$L(x;a,b)$	0.0049	0.0696	
$h_{[25]}$	$L(x;a,b)$	0.0136	−0.1485	
$h_{[30]}$	$L(x;a,b)$	−0.0031	0.3545	
$h_{[35]}$	$L(x;a,b)$	0.007		

Pandey's data (1972) was used to compute the parameters of each membership function.

2.2. Parametrization of the model

The model has three parameters which are the length of the newly hatched larva (l_0), the length of the L3 larva (l_{L3}) and the development rate (r), all of which depend on the daily average temperature. Each of these parameters is modelled using a fuzzy rule-based system of the Takagi-Sugeno-Kant type (FTSK) (see Appendix A), with the temperature as the input variable.

The construction of the membership functions depending on temperature was based on the work of Pandey (1972), who investigated the effect of temperature (between 4 °C and 40 °C) on the development of larvae in the free-living stage. The "temperature" variable (Temp) is partitioned into six membership functions, named respectively Temp10, Temp15, Temp20, Temp25, Temp30, and Temp35 (i.e. Temp15 corresponding to temperatures within an interval centered at 15 °C). In each case, the maximum value of membership is coincident with that in Pandey's work. These parameters are detailed in Table 1.

Data on the length of newly hatched larvae, the length of the infective larvae and development times were used to build the consequent function of the FTSK system.

2.2.1. Length of newly hatched larvae $l_0(T_t)$

A wide variation in the size of the newly hatched larvae was observed at different temperatures, the smallest larvae being obtained at 35 °C and the largest at 15 °C (Pandey, 1972). As mentioned earlier, $l_0(T_t)$ is a FTSK system with "temperature" as the input variable. The consequent functions are constructed according to Pandey's data, which allow the possibility of locally describing the dynamics of the problem in approximate terms.

This means that, for example, if the lengths of newly hatched larvae given by Pandey are $l_0(10°) = 383$ at 10 °C and $l_0(15°) = 413$ at 15 °C, then for intermediate temperature values a linear functional relationship is assumed, the function being $f_{[10]}(x_1) = 6x_1 + 323$. Then, the fuzzy rule for $x_1 \in$ Temp10 is:

$$\text{If } x_1 \text{ is Temp10} \Rightarrow l_0(x_1) \text{ is } f_{[10]}(x_1) = 6x_1 + 323$$