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A dynamic programming implemented resource competition game theoretic model

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ABSTRACT

Resource competition is commonly occurred in animal populations and studied intensively by researchers. Previous studies have applied game theoretic model by finding Nash equilibrium to investigate this phenomenon. However computation of the Nash equilibrium requires an understanding of the payoff matrix that allocates the rewards received by players when they adopt each of the strategies in the game. In our study we present a dynamic programming implemented framework to compute 2×2 intraspecific finite resource allocation game's payoff matrix explicitly. We assume that two distinct types of individuals, aggressive and non-aggressive, are in the population. Then we divide the entire animal development period into three different stages: initialization, quasilinear growth and termination. Each stage for each type of players is specified with their own development coefficient, which determines how resource consumption could convert into strength as reward. Each player has equal and finite resource at the beginning of their development and fights against other players in the population to maximize its own potential reward. Based on these assumptions it is reasonable to use backward induction dynamic programming to compute payoff matrix. We present numerical examples for three different types of aggressive individuals and compute the payoff matrices correspondingly. Then we use the derived payoff matrices to determine the Nash equilibrium and Evolutionary Stable Strategy. Our research provide a framework for future quantitative studies on animal resource competition problems and could be expanded to n-players interspecific stochastic asymmetric resource allocation problem by changing some settings of dynamic programming formulation.

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1. Introduction

The initial concept of Game Theory was developed by Neumann and Morgenstern (1944). Later Nash made significant contribution to Game Theory by introducing the idea of Nash equilibrium. If each player in the game chooses a strategy and no player can benefit by changing its strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute Nash equilibrium. Nash also proved every finite game has mixed Nash equilibria and this idea has become the keystone of Game Theory (Nash, 1950). Since then Game Theory has been widely applied in various disciplines such as social sciences, economics, political science, international relationship, computer science and philosophy (Osborne, 2004). Smith and Price (1973) formalized another central concept in Game Theory called the Evolutionary Stable Strategy (ESS), which, if chosen by a population of players, cannot be invaded by any alternative strategy that is initially rare. Various studies have been done using ESS to investigate animal competition behavior and evolutionary path of them. Crowley (2000) studied the hawk-dove game in symmetry payoff matrix and determined its ESS. Tainaka and Itoh (2002) have studied cooperation and altruism in Prisoner's Dilemma. Matsumura and Hayden (2006) studied animal communication using ESS model. Wolf and Mangel (2007) analyzed compromise and cheating in predator-prey games. Hamblin and Hurd (2009) also studied deceptive signaling during the games.

The simplest and most common case of game has 2×2 non-cooperative form. Multiple players game is more realistic but according to Poincare–Bendixson theorem (Strogatz, 2001), multiple players dynamic system would result in chaos and therefore become relatively difficult to analyze. In our study we adopt 2×2 games. While the key concept in Game Theory is Nash equilibrium, we should achieve the payoff matrix very carefully in order to compute Nash equilibrium correctly. However, most of the research studies regarding ESS have arbitrarily assigned values in payoff matrix. To overcome this problem, we want to use some more realistic quantities to determine payoff matrix for the game. Mesterton-Gibbons and Sherratt (2009) has shown a neighbor intervention model and Luther et al. (2007) further discussed whether food is worth fighting for during the game. Just et al. (2007)

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Table 1
Payoff matrix of non-cooperative general sum game.

Strategy	Non-aggressive	Aggressive
Non-aggressive	(P_{111}, P_{112})	(P_{121}, P_{122})
Aggressive	(P_{211}, P_{212})	(P_{221}, P_{222})

P_{ijk} denotes the payoff of player k when it uses strategy i and its competitor uses j.

has studied the payoff gained by aggressive player in a competitive game. Followed by these ideas, we consider food resource consumed by the animal might be a measurement to compute the payoff matrix. However it ignores individual difference and we consider using a function that links resource consumption and potential body strength, the reward, to compute the payoff.

While we assume the food resource is finite, initially equivalent to any player in the population, and animals have discrete development stages, it is reasonable to use dynamic programming (DP) to figure out the optimal foraging strategy as a resource allocation problem. DP is a method of solving complex problems by breaking them down into simpler steps. Kaewmanee and Tang (2003) used an age-structured model to define cannibalism. Chambon-Dubreuil et al. (2006) further investigated the effect of aggressive behavior on age-structured populations. We treat animal growth as a logistic function and divide the development into three distinct stages: initialization, quasilinear growth and termination. In different stages, we assume the reward is a unique linear function of food resource. Our goal is to determine the maximum total reward at the end of growth using DP to determine the payoff value, which is rarely applied in ecological research (Frelek, 1982).

Al-Tamimi et al. (2007) have suggested using dynamic programming to implement Game Theory model for designing problems. However their model is based on zero-sum game. In our study we will first present a more realistic general sum game framework by using DP to compute payoff matrix, discuss three different types of aggressive players and calculate the numerical payoff matrix for each case and determine the ESS for them, respectively. Our work is the first of this kind to combine DP and Game Theory, two different optimization tools together to solve real biological problems.

2. The model

A typical 2×2 non-cooperative general sum game has the form shown in Table 1, where P_{ijk} represents the payoff of player k when it uses strategy i and its competitor uses the alternative strategy j. Here we have two types of strategies: aggressive (2) and non-aggressive (1). Aggressive players would fight their neighbor and try to seize the neighbor's resources. Non-aggressive players only concentrate on their own resource and never fight back even attacked by aggressive players. However, two aggressive players would result in a severe fight and both players would be hurt intensively. This setting is similar to that of "Chicken-Dare" or "Hawk-Dove" game (Osborne, 2004). Mathematically, Nash equilibrium in this game is defined as:

$X \in \Theta$ is a Nash equilibrium if $X \in \beta(x)$ where Θ is the mixed strategy space and β is the mixed strategy best response correspondence. Because this is a 2×2 finite symmetric game $\Delta^{NE} \neq \varnothing$.

From a population perspective we could define Evolutionary Stable Strategy (ESS) as follows:

$X \in \Delta$ is an ESS if for every strategy $Y \neq X$ there exists some $\varepsilon_y \in (0, 1)$ such that $u[X, \varepsilon Y + (1-\varepsilon)X] > u[Y, \varepsilon Y + (1-\varepsilon)X]$ holds for all $\varepsilon \in (0, \varepsilon_y)$ where ε is the proportion of mutant strategy.

Basically, ESS is a subset of Nash equilibrium. We use Maynard's criterion (Maynard, 1973) to test whether Nash equilibrium is an ESS: $\Delta^{ESS} = \{X \in \Delta^{NE} : u(Y, Y) < u(X, X), \forall Y \in \beta(X), Y \neq X\}$.

To perform all these analyses, we first compute the payoff matrix in our original game. We will use DP to determine the numerical payoff values for the four strategy combinations. Assume each player has a total of N units of resources for the entire development period initially and in each stage at least 1 resource should be consumed in order to maintain basal metabolism. As we have discussed before, the entire development is divided into three stages: growth initialization, quasilinear growth and growth termination, hence the player could possibly consume 1 to $N-2$ units of resources in each stage. While the growth is logistic and nonlinear, we could use linear approximation in each stage as follows where y is the reward in each stage and x is the number of units of resources consumed:

$$y = \begin{cases} ax(\text{growth initialization}) \\ bx(\text{quasilinear growth}) \\ cx(\text{growth termination}) \end{cases}$$

Because logistic curve has a sigmoid shape and is usually symmetric, it is reasonable to set $a = c$ to reduce computational intensity. The coefficients a and b have biological meaning that they determine the efficiency of converting resources into the animal's own energy or strength. In our model we assume $b > a$ because of their development characteristics. The DP model is written as follows where z is total reward from food resource that we want to maximize, i is development stage, x_i is the amount of resources consumed at stage i, N is total amount of resources, r_i is the conversion coefficient and $r_1 = a$, $r_2 = b$ and $r_3 = a$:

$$\text{Maxmize}\, z = \sum_{i=1}^{3} r_i x_i$$

$$\text{Subject to} \sum_{i=1}^{3} x_i = N$$

We use backward induction, a solution method for finite-horizon discrete-time dynamic optimization problems, to solve this problem. The backward induction formulation is given as follows (Howard, 1960):

Objective Value Function: $f_i(x)$ = optimal reward given x units of resource are to be allocated at stage i, $i = 1, 2, 3$.
Argument: (i, x) = (stage, units of resource consumed).
Recurrent relation: $f_i(x) = \max_{x_i=1,2\ldots N-2}[r_i x_i + f_{i+1}(x - x_i)]$, by principle of optimality (Bellman, 1952).
Boundary condition: $f_3(x) = r_3 x_3$.
Answer: $f_1(N)$. This specifies when the backward induction should terminate.

For the non-aggressive and non-aggressive strategy combination, we assume both players do not interfere with each other. In this case, we would only solve the DP for either one of them and by symmetry the other player should adopt the same strategy to maximize its total reward. The reward in each stage and state is shown in Table 2 and we could calculate the optimal value using DP.

For the non-aggressive and aggressive strategy combination, the reward table is similar to Table 2. The difference is we should define different resource-reward converting coefficients, a and b for both strategies. For the aggressive and aggressive combination we will also define the coefficients. Once we have computed the rewards for each combination we could construct the payoff matrix, calculate

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