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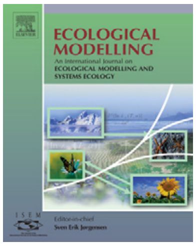

Defining yield policies in a viability approach

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ABSTRACT

Mullon et al. [Mullon, C., Curry, P., Shannon, L., 2004. Viability model of trophic interactions in marine ecosystems. Nat. Resour. Model. 17 (1), 27–58] proposed a dynamical model of biomass evolution in the Southern Benguela ecosystem, including five different groups (detritus, phytoplankton, zooplankton, pelagic fish and demersal fish). They studied this model in a viability perspective, trying to assess, for a given constant yield, whether each species biomass remains inside a given interval, taking into account the uncertainty on the interaction coefficients. Instead of studying the healthy states of this marine ecosystem with a constant yield, we focus here on the yield policies which keep the system viable. Using the mathematical concept of viability kernel, we examine how yield management might guarantee viable fisheries. One of the main practical difficulties up to now with the viability theory was the lack of methods to solve the problem in large dimensions. In this paper, we use a new method based on SVMs, which gives this theory a larger practical potential. Solving the viability problem provides all yield policies (if any) which guarantee a perennial system. We illustrate our main findings with numerical simulations.

1. Introduction

The viability theory (Aubin, 1991) aims at controlling dynamical systems with the goal to maintain them inside a given set of admissible states, called the viability constraint set. Such problems are frequent in ecology or economics, when systems die or badly deteriorate if they leave some regions of the state space. For instance Bene et al. (2001) studied the management of a renewable resource as a viability problem. They pointed out irreversible overexploitation related to the resource extinction. Bonneuil (2003) studied the conditions the prey-predator dynamics must satisfy to avoid extinction of one or the other species as a viability problem. Cury et al. (2005) consider viability theory to advise fisheries.

Mullon et al. (2004) proposed a dynamical model of biomass evolution of the Southern Benguela ecosystem, involving five different groups (detritus, phytoplankton, zooplankton, pelagic fish and demersal fish). They studied this model in a viability perspective (Aubin, 1991), trying to assess, for a given constant yield, whether each species biomass remains inside a given interval, taking into account the uncertainty on the interaction coefficients. The aim was to identify constant yield values that allow persistence of the ecosystem. We extend the problem and we focus here on the yield policies which keep the system viable, instead of considering a constant yield.

Using the mathematical concept of *viability kernel*, we examine how yield management might guarantee viable fisheries. The *viability kernel* designates the set of all viable states, i.e. for which there exists a control policy maintaining them within the set of constraints. Outside the viability kernel, there is no evolution which prevents the system from collapsing. Aubin (1991) proved the viability theorems which enable to determine the viability kernel, without considering the combi-

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natorial exploration of control actions series. These theorems also provide the control functions that maintain viability.

This general approach shows several interesting specific aspects:

- It can take into account the uncertainties on the parameters which are generally high in ecosystem modelling. Here, we manage the uncertainties like in Mullon et al. (2004).
- The viability kernel can define a variety of different policies, which respect the viability constraints. Therefore, it offers more possibilities for negotiations and discussions among the concerned stakeholders than techniques which propose a single optimal policy.

The main limitation of the viability approach is its computational complexity. The existing algorithm for viability kernel approximation (Saint-Pierre, 1994) supposes an exhaustive search in the control space at each time step. This makes the method impossible to use when the control space is of a 51 dimensions like in our problem. Mullon et al. (2004) solved this problem with a method which is only adapted to linear equations of evolution. Here, we use a new method, based on support vector machines, which can be applied to non-linear models as well (Deffuant et al., 2007).

We present the viability model of the Southern Benguela ecosystem and we recall the main concepts of the viability theory. Then, we describe our main numerical results. We show the shape of the found viability kernel, and the corresponding possible yield policies. Finally, we discuss the results and draw some perspectives.

2. The viability model of the Southern Benguela ecosystem

Following a classical approach (Walters and Pauly, 1997), we suppose that the variation of the biomass of species *i* due to its predation by other species *j* depends linearly on the recipient and donor biomasses (B_j and B_i), with respective coefficients r_{ji} and d_{ji}. The biomass lost by species *i* due to the predation by the other species is expressed by Eq. (1):

$$\frac{\mathrm{d}B_i(i \rightarrow)}{\mathrm{d}t} = -\sum_j (r_{ji}B_j + d_{ji}B_i). \tag{1}$$

The variation of the donor biomass B_i due to this interaction takes into account the assimilation of the biomass of other species *j*, multiplied by a growth efficiency coefficient (denoted below by g_i). Therefore, the biomass gained by species *i*, because of its consumption of other species, is expressed by:

$$\frac{\mathrm{d}B_i(i \leftarrow)}{\mathrm{d}t} = g_i \sum_j (r_{ij}B_i + d_{ij}B_j). \tag{2}$$

For the detritus, the variation of the biomass follows the same principle, but it also integrates the non-assimilated biomass of the other species, except phytoplankton, which is added to the detritus biomass B_1 (multiplied by its growth efficiency g_1):

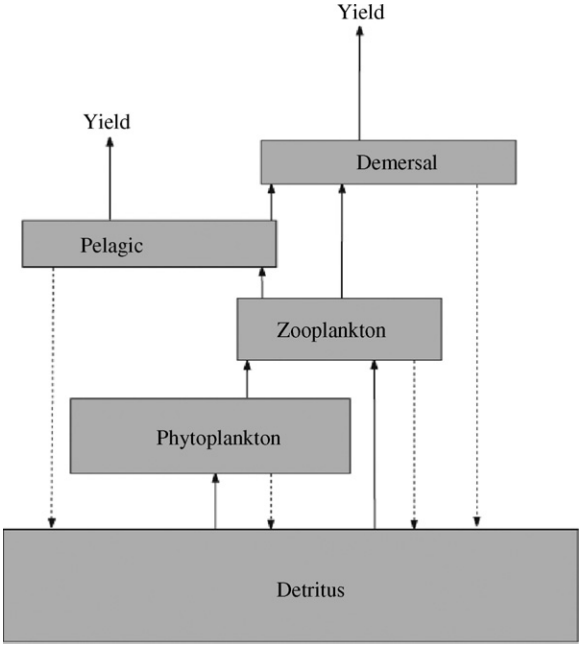

Fig. 1 – Components and structure of the Southern Benguela ecosystem. Arrows represent the flux between compartments (from Mullon et al., 2004).

$$\frac{\mathrm{d}B_1(\text{non-assimilated})}{\mathrm{d}t} = \sum_{j>2}\sum_k g_1(1-g_l)(r_{lk}B_l + d_{kl}B_k). \tag{3}$$

The model of the Southern Benguela ecosystem considers trophic interactions (predation, consumption and catch) among five components: detritus (i = 1), phytoplankton (i = 2), zooplankton (i = 3), pelagic fish (i = 4), demersal fish (i = 5). In total, the biomass evolution can be written as follows:

$$\begin{aligned}
\frac{\mathrm{d}B_1}{\mathrm{d}t} &= \frac{\mathrm{d}B_1(1 \leftarrow)}{\mathrm{d}t} - \frac{\mathrm{d}B_1(1 \rightarrow)}{\mathrm{d}t} + \frac{\mathrm{d}B_1(\text{non-assimilated})}{\mathrm{d}t} - Y_1,\\
\frac{\mathrm{d}B_i}{\mathrm{d}t} &= \frac{\mathrm{d}B_i(i \leftarrow)}{\mathrm{d}t} - \frac{\mathrm{d}B_i(i \rightarrow)}{\mathrm{d}t} - Y_i,
\end{aligned} \tag{4}$$

where g_i is the growth efficiency of species i, Y_i is the yield of species i. Fig. 1 shows the structure of the ecosystem.

Mullon et al. (2004) take into account the uncertainty on parameters r_{ij} and d_{ij}, which is expressed by:

$$r_{ij} \in \left[\bar{r}_{ij} - \delta r_{ij}, \bar{r}_{ij} + \delta r_{ij}\right], \qquad d_{ij} \in \left[\bar{d}_{ij} - \delta d_{ij}, \bar{d}_{ij} + \delta d_{ij}\right]. \tag{5}$$

They consider this model in a viability perspective, in order to study the persistence of the ecosystem and to define the impact of the fisheries. Given a constant yield, they define scenarios which result is a “healthy” system.

Extending the work of (Mullon et al., 2004), we incorporate the fisheries in this study as a control variable of the system, in order to find the yield policies which allow keeping the system viable. To guarantee a perennial system, the viability