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Preemptive scheduling on identical machines with delivery coordination to minimize the maximum delivery completion time

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ABSTRACT

In this paper, we consider a two-stage scheduling problem on identical machines in which the jobs are first processed preemptively on m identical machines at a manufacturing facility and then delivered to their customers by one vehicle which can deliver one job at each shipment. The objective is to minimize the maximum delivery completion time, i.e., the time when all the jobs are delivered to their respective customers and the vehicle returns to the facility. We first show that the problem is strongly NP-hard. We then present a $\frac{3}{2}$-approximation algorithm and show that the bound is tight.

1. Introduction

This paper studies a two-stage scheduling problem in which the first stage is job production and the second stage is job delivery. Modern business market has been more and more competitive. In order to be competitive, the companies have to reduce their storage costs. That is, all jobs are needed to be transported as soon as possible to another machine for further processing or to their customers. Due to its importance in manufacturing industry, machine scheduling with delivery coordination has been widely studied over the last twenty years. According to the transportation function, problems on this topic can be classified into two types (see Lee and Chen [12]). The first type (type-1) involves intermediate transportation of the unfinished jobs from one machine to another for further processing. The second type (type-2) involves outbound transportation of the finished jobs from the machine(s) to their customer(s).

The earliest scheduling paper with type-1 transportation is the one studied by Maggu and Das [17]. They studied a two-machine flow-shop scheduling problem to minimize the makespan. In this problem, both of the two machines have unlimited buffers and there are enough vehicles to transport jobs from the first machine to the other. They solved the problem by using a generalization of Johnson's rule [10]. Maggu et al. [18] considered the same problem but imposed additional constraints on the sequence of the processing jobs. Kise [11] studied a similar problem in which there is only a vehicle and the vehicle can transport a job at a time. Stern and Vitner [24], Ganesharajah et al. [3], Panwalkar [19], and

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Gemmill and Stevens [6] considered a similar problem with limited buffer spaces and unit vehicle capacity. Hurink and Knust [7] studied independently a similar problem in which the return time of the vehicle is zero.

The earliest scheduling paper with type-2 transportation is the one studied by Potts [20]. He studied a single machine scheduling problem with unequal job arrival times and delivery times to minimize the makespan. In this problem, it is assumed that there are a sufficient number of vehicles so that each finished job can be delivered individually and immediately to its customer. The author presented a heuristic algorithm with the worst-case performance ratio of $\frac{3}{2}$ for the problem. Hall and Shmoys [8] presented two polynomial-time approximation schemes for the same problem. Woeginger [25] studied the same problem in the parallel-machine environment with equal job arrival times. Strusevich [21] considered a two-machine open-shop scheduling problem. For this problem, the author presented a heuristic algorithm with the worst-case performance ratio of $\frac{3}{2}$.

Lee and Chen [12] studied several scheduling problems with type-2 transportation. However, in their problems, there are v vehicles with the same capacity c to transport all finished jobs. Thus, when a job completes its processing, it has to wait for some time until some vehicle becomes available. Soukhal et al. [22] and Yuan et al. [28] studied the two-machine flow-shop scheduling problem to minimize the makespan. They showed that the problem is binary NP-hard when $c = 2$ and is strongly NP-hard when $c \geq 3$ even if the jobs have an equal processing time on the first machine and all jobs have equal transportation time. Lu et al. [15] considered the single-machine scheduling with release dates in which only a vehicle can be used to deliver all jobs to a single customer. They showed that the problem is strongly NP-hard for each fixed $c \geq 1$ and gave a heuristic with a tight worst-case performance ratio of $\frac{5}{3}$.

Wang and Cheng [26] considered the scheduling problems with an unavailable interval on the machine(s). For the single machine scheduling problem, they showed that this problem is NP-hard and proposed a heuristic with a tight worst-case performance ratio of $\frac{3}{2}$. For the two parallel machines scheduling problem, they proposed a heuristic with a worst-case performance ratio of $\frac{5}{3}$. Wang and Cheng [27] further studied a single-machine scheduling problem in which there is a capacitated vehicle to transport unprocessed jobs from the supplier's warehouse to the factory and another capacitated vehicle to deliver finished jobs to the customer. They showed that this problem is NP-hard in the strong sense and proposed a heuristic with a tight performance ratio of 2. Dong et al. [2] considered a two-machine open-shop problem with one customer. They gave two algorithms with worst-case performance ratios of 2 and $\frac{3}{2}$ when $c \geq 2$ and $c = 1$, respectively.

Chang and Lee [1] extended Lee and Chen's model in [12] by considering the situation where each job might occupy a different amount of physical space in a vehicle. They provided a heuristic with a worst-case performance ratio of $\frac{5}{3}$ for the single machine scheduling problem and a heuristic with a worst-case performance ratio of 2 for the two parallel machines scheduling problem. For the single machine scheduling problem, He et al. [9] presented an improved approximation algorithm with a worst-case performance ratio of $\frac{53}{35}$. For the same problem, Lu and Yuan [13] provided a heuristic with the best-possible worst-case performance ratio of $\frac{3}{2}$. Lu and Yuan [14] also extended Chang and Lee's problem in [1] on an unbounded parallel-batch machine. They showed that the problem is strongly NP-hard and gave a heuristic with a worst-case performance ratio of $\frac{7}{4}$. For the two parallel machines scheduling problem, Zhong et al. [29] presented an improved algorithm with a worst-case ratio of $\frac{5}{3}$ and Sua et al. [23] proposed a heuristic with a worst-case performance ratio of $\frac{63}{40}$, except for two particular cases.

In this paper, we consider a two-stage scheduling problem in which a set $N = \{1, 2, \cdots, n\}$ of n jobs are first processed preemptively on m identical machines and then delivered to their customers by only one vehicle which can deliver one job at each shipment. A schedule for the problem includes a scheme for the preemptive processing of the n jobs on the m machines and a scheme for the delivery of the n jobs, where a job j can be delivered only if it has completed its processing and the only vehicle is available. The objective is to minimize the maximum delivery completion time, i.e., the time when all the jobs are delivered to their respective customers and the vehicle returns to the facility. Let D_j be the delivery completion time of job j, i.e., the time when job j is delivered to its customer and the vehicle returns to the facility. We use $D_{\max} = \max\{D_j : j \in N\}$ to denote the maximum delivery completion time of all jobs. Following the classification scheme for scheduling problems by Graham et al. [5], the problem in consideration is denoted by $P|\text{pmtn}|D_{\max}$. We show in this paper that the problem is strongly NP-hard and present a $\frac{3}{2}$-approximation algorithm.

The paper is organized as follows. In Section 2, we provide some useful notations and lemmas. The NP-hardness proof is proposed in Section 3 and the approximation algorithm is presented in Section 4.

2. Preliminaries

The following notations are used in this paper.

- $N = \{1, 2, \cdots, n\}$ is the set of n jobs to be scheduled.
- p_j is the processing time of job j.
- q_j is the round-trip delivery time of job j between the machine and the customer.
- $p_{\max}(J) = \max\{p_j : j \in J\}$ is the maximum processing time of the jobs in $J \subseteq N$.
- $p_{\min}(J) = \min\{p_j : j \in J\}$ is the minimum processing time of the jobs in $J \subseteq N$.
- $q_{\max}(J) = \max\{q_j : j \in J\}$ is the maximum delivery time of the jobs in $J \subseteq N$.
- $q_{\min}(J) = \min\{q_j : j \in J\}$ is the minimum delivery time of the jobs in $J \subseteq N$.