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Pareto optimization scheduling of family jobs on a p-batch machine to minimize makespan and maximum lateness

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ABSTRACT

This paper studies the Pareto optimization scheduling problem of family jobs on an unbounded parallel-batching machine to minimize makespan and maximum lateness. In the problem, the jobs are partitioned into families and scheduled in batches, where each batch is a set of jobs belonging to a common family and the processing time of a batch is defined to be the longest processing time of the jobs in the batch. The objective is to find all Pareto optimal points for minimizing makespan and maximum lateness and, for each Pareto optimal point, provide a corresponding Pareto optimal schedule. We present an algorithm to solve this Pareto optimization problem. Our algorithm is of polynomial-time when the number of families is a constant.

1. Introduction

A parallel-batching (shortly, p-batch) machine [11] is a machine which can process the jobs in (parallel) batches, where each batch is a set of jobs, the processing time of a batch is the longest processing time of the jobs in the batch, i.e., $p(B) = \max\{p_j : J_j \in B\}$, and the completion time of a job is the completion time of the batch containing the job. Let b be the batch capacity. Then each batch can contain at most b jobs. Let n be the number of jobs to be scheduled. When $b < n$, we call the setting as bounded p-batch, and when $b \ge n$, we call the setting as unbounded p-batch. When all jobs are available at time 0 and the objective function to be minimized is regular (nondecreasing in the completion time of the jobs), a schedule can be given by a batch sequence $\sigma = (B_1, B_2, \cdots, B_k)$ which presents the processing order of the batches scheduled consecutively. Then the completion time of a job $J_j \in B_i$ in σ is given by $C_j(\sigma) = p(B_1) + p(B_2) + \cdots + p(B_i)$.

Following Hoogeveen [6], the Pareto optimization scheduling problem on a single machine to minimize two objective functions f and g can be formulated as follows:

For a given schedule π, we denote by $(f(\pi), g(\pi))$ the objective vector of π. If there exists no schedule σ such that $(f(\sigma), g(\sigma)) \le (f(\pi), g(\pi))$ and at least one of the two strict inequalities $f(\sigma) < f(\pi)$ and $g(\sigma) < g(\pi)$ holds, we call π a *Pareto optimal schedule* and $(f(\pi), g(\pi))$ a *Pareto optimal point* corresponding to π. The goal of the Pareto optimization scheduling is to find all Pareto optimal points and, for each Pareto optimal point, provide a corresponding Pareto optimal schedule. Following the three-parameter scheduling notation introduced by Graham et al. [4], the Pareto optimization scheduling problem on a single machine to minimize two objective functions f and g can be denoted by $1|\beta|f \circ g$, where β denotes the restricted constraints of the feasible schedules. Related to problem $1|\beta|f \circ g$, there are two constrained optimization scheduling problems $1|\beta|f : g \le U$ and $1|\beta|g : f \le V$.

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Suppose that we have n family jobs $J_1, J_2, \cdots, J_n$ to be processed on a parallel-batching machine with capacity $b \ge n$. In our research the jobs are from F different families $\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_F$ which form a partition of $\{J_1, J_2, \cdots, J_n\}$. For each f with $1 \le f \le F$, we write $\mathcal{F}_f = \{J_{f,1}, J_{f,2}, \cdots, J_{f,n_f}\}$ and $n_f = |\mathcal{F}_f|$. The jobs from different families are not permitted to be processed together in any single batch, and we say that these families are *incompatible*. Each job J_j $(J_{f,j})$ has a processing time p_j $(p_{f,j})$ and a due date d_j $(d_{f,j})$. In this paper we assume that p_j $(p_{f,j})$ and d_j $(d_{f,j})$ are positive integers and all the jobs are available for processing in batches from time zero onwards.

For a given schedule (batch sequence) $\sigma = (B_1, B_2, \cdots, B_k)$, let $C_j(\sigma)$ denote the completion time of job J_j and $C_{\max}(\sigma) = \max_{1 \le j \le n} C_j(\sigma)$ denote the makespan of σ (i.e., the maximum completion time of the jobs in σ). The lateness of job J_j in σ is defined as $L_j(\sigma) = C_j(\sigma) - d_j$ and the maximum lateness of σ is given by $L_{\max}(\sigma) = \max_{1 \le j \le n} L_j(\sigma)$. We study the Pareto optimization scheduling to minimize two objective functions $C_{\max}$ and $L_{\max}$. Using the standard three-field $\alpha|\beta|\gamma$ notation, we denote the investigated Pareto optimization scheduling problem by $1|\text{p-batch}, \text{family-jobs}, b \ge n|C_{\max} \circ L_{\max}$, where "p-batch" means the parallel-batching setting, "family-jobs" means that the job families are incompatible, and "$b \ge n$" means the unbounded batch capacity. For convenience, we shortly use "β" to denote "$\text{p-batch}, \text{family-jobs}, b \ge n$". Then the Pareto optimization problem is simply denoted by $1|\beta|C_{\max} \circ L_{\max}$. Furthermore, the two related constrained optimization problems are denoted by $1|\beta|C_{\max} : L_{\max} \le L$ and $1|\beta|L_{\max} : C_{\max} \le C$.

The p-batch scheduling model was first introduced in Lee et al. [11] with bounded capacity. The research of Lee et al. [11] was motivated by the burn-in operations during the final testing stage of circuit board manufacturing. There are many other applications of p-batch scheduling, such as the diffusion area in semiconductor wafer fabrication facilities, heat treatment facilities in the steel and ceramic industries, the numerically controlled routers for cutting metal sheets or printed circuit boards, etc, see [3,13,16]. A complete complexity classification on p-batch scheduling can be referred to [1,2,15]. P-batch scheduling with job families have been studied by several researchers in the literatures. Yuan et al. [19] considered the scheduling problem on a single unbounded p-batch machine with family jobs and release dates to minimize makespan. They showed that the problem is strongly NP-hard and presented two dynamic programming algorithms and a heuristic of a performance ratio 2. Nong et al. [17] and Li et al. [14] considered the single machine bounded p-batch setting and parallel-machine p-batch setting for the problem studied in Yuan et al. [19], respectively. More research on p-batch scheduling with job families can be found in [9,12,13,16].

For the research of Pareto optimization scheduling, we can refer to the survey [6] for detailed developments. We only mention two related results here. Hoogeveen [7] showed that the Pareto optimization scheduling problem of minimizing two maximum cost criteria, i.e., $1||f_{\max} \circ g_{\max}$, is solvable in $O(n^4)$ time. He et al. [5] studied the scheduling problem $1|\text{p-batch}, b \ge n|C_{\max} \circ L_{\max}$ and presented an $O(n^3)$-time algorithm.

The problem $1|\beta|C_{\max} \circ L_{\max}$ studied in this paper is a generalization of problem $1|\text{p-batch}, b \ge n|C_{\max} \circ L_{\max}$ studied in He et al. [5]. Note that the complexity of problem $1|\text{p-batch}, \text{family-jobs}, b \ge n|L_{\max}$ is equivalent to the complexity of problem $1|r_j, \text{p-batch}, \text{family-jobs}, b \ge n|C_{\max}$ which have been proved to be strongly NP-hard when the number of families is arbitrary in Yuan et al. [19]. Then our problem $1|\beta|C_{\max} \circ L_{\max}$ is also strongly NP-hard.

In our research, we first present a dynamic programming Algorithm DP(L) for the constrained optimization problem $1|\beta|C_{\max} : L_{\max} \le L$ with time complexity $O(n^{F+1})$. Algorithm DP(L) returns an optimal schedule for problem $1|\beta|C_{\max} : L_{\max} \le L$ which is also a Pareto optimal schedule for problem $1|\beta|C_{\max} \circ L_{\max}$. By investigating the properties of Algorithm DP(L), we present an improved Algorithm DP*(L) with time complexity $O(n^F)$. Then the Pareto optimal points and their corresponding Pareto optimal schedules can be iteratively generated by using Algorithm DP*(L). We introduce a new technique for counting the number of the Pareto optimal points, and show that problem $1|\beta|C_{\max} \circ L_{\max}$ has at most $\frac{n}{2} \cdot \prod_{1 \le f \le F}(n_f + 1) = O(n^{F+1})$ Pareto optimal points. As a result, problem $1|\beta|C_{\max} \circ L_{\max}$ can be solved in $O(n^{2F+1})$-time algorithm which is of polynomial-time when the number F of families is a constant.

The remainder of this paper is organized as follows. In Section 2, we present some notations and basic lemmas. In Section 3, we describe our algorithm for the problem $1|\beta|C_{\max} \circ L_{\max}$ together with the analysis.

2. Preliminaries

Consider the Pareto optimization scheduling problem $1|\bullet|f \circ g$ on a single machine to minimize two objective functions f and g. A schedule π is called an *optimal and Pareto optimal schedule* of the constrained problem $1|\bullet|f : g \le V$ if π is optimal for problem $1|\bullet|f : g \le V$ and also Pareto optimal for problem $1|\bullet|f \circ g$. The following lemma in Hoogeveen [6] presented a method to find an optimal and Pareto optimal schedule.

Lemma 2.1. *(See [6].) Let V be a threshold value so that problem $1|\bullet|g \le V$ is feasible. Suppose that x is the optimal value of problem $1|\bullet|f : g \le V$, and y is the optimal value of problem $1|\bullet|g : f \le x$. Then (x, y) is a Pareto optimal point of problem $1|\bullet|f \circ g$, and furthermore, every optimal schedule of problem $1|\bullet|g : f \le x$ is an optimal and Pareto optimal schedule of problem $1|\bullet|f : g \le V$ corresponding to (x, y).* □

If $\mathcal{A}(V)$ is an algorithm which either claims that problem $1|\bullet|g \le V$ is infeasible or returns an optimal and Pareto optimal schedule of problem $1|\bullet|f : g \le V$, then we say that $\mathcal{A}(V)$ is an *optimal and Pareto optimal algorithm* for problem $1|\bullet|f : g \le V$. With an optimal and Pareto optimal algorithm for problem $1|\bullet|f : g \le V$ in hand, the Pareto optimization scheduling problem $1|\bullet|f \circ g$ can be solved by the following algorithm Pareto-Generating($\mathcal{A}(V)$) iteratively.