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Improved algorithms for single-machine common due window assignment and scheduling with batch deliveries

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ABSTRACT

We consider a single-machine due window assignment and scheduling problem with batch deliveries, where all jobs have a common due window, and the start time and size of the due window are decision variables. Finished jobs are delivered in batches with unlimited batch capacity. The objective is to determine the due window, a job sequence, and the delivery times, so as to minimize the total cost which comprises earliness of delivery, job holding, start time of due window, size of due window, number of delivery batches, and tardiness penalty. We consider three different variants of the problem corresponding to different measurements of tardiness penalty. We present polynomial-time solution procedures for these variants with significantly lower computational complexities than those of known algorithms in the literature.

1. Introduction

Due window assignment research extends the traditional "due date assignment" research in such a way that it becomes essential for a manufacturer to determine a due time window instead of a single due date for his jobs. Due window assignment decisions often occur when negotiations of due windows between the manufacturer and his customers take place (Janiak and Winczaszek [9]). When deciding the due window for a group of jobs, the manufacturer has to take into account the size of the due window, because a small due window will decrease the flexibility of his internal operations and may result in an excessive number of early and tardy jobs, while a large due window provides little delivery information to the customers and makes them difficult to plan their own operations. The manufacturer must also determine the start time of the due window carefully, as an early due window start time will increase the amount of job tardiness, while a late start time will make the delivery schedule unattractive to the customers. The manufacturer also needs to determine the production schedule of the jobs concerned. In practice, finished jobs are often delivered in batches, and the due window can be viewed as a delivery time window of the finished jobs. However, batching decisions often increase the complexity of the due window assignment and scheduling problem because the manufacturer needs to group the jobs into batches and determine the delivery time of each batch.

Due window assignment and scheduling problems have been widely studied in the last two decades. Most research on this topic involves the determination of a job schedule as well as a common due window for all jobs concerned. Problems with different machine configurations (e.g., single machine, parallel machines, etc.) and different cost measurements have been studied, with many works incorporating features such as controllable job processing times, deteriorating jobs, flow

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allowance, etc., into the due window assignment models. Janiak et al. [8] offered a comprehensive review on the computational complexity results of common due window assignment problems. Recently, Gerstl and Mosheiov [4] examined a class of common due window assignment problems with unit processing time jobs and developed polynomial-time algorithms for these problems. In another study, a common due window assignment problem with identical processing time jobs on uniform machines was examined (Gerstl and Mosheiov [5]). Janiak et al. [7] considered a common due window assignment problem on parallel machines with nonlinear cost functions. Wang et al. [15] analyzed a due window assignment problem with learning effects and deteriorating jobs, where the due windows are job-dependent. They considered two due window assignment methods, namely the slack due date assignment method and the unrestricted due date assignment method. Yang et al. [16] investigated a multiple due windows assignment problem with controllable job processing times, where one of the decisions is to assign jobs to a predetermined number of due windows. Ji et al. [10] studied a due window assignment problem, in which the jobs are processed in groups, and a setup time is associated with the processing of the first job of each group. In their model, the due windows are job-dependent but all jobs have a common flow allowance. Yin et al. [20] considered a due window assignment problem with controllable job processing times, where the due windows are job-dependent but all the jobs have a common flow allowance. A number of recent studies have examined models that combine due window assignments and the scheduling of maintenance activities with improved processing times after maintenance works are carried out. These include the works of Mosheiov and Sarig [14], Yang et al. [17], Zhu et al. [23], Cheng et al. [3], Mor and Mosheiov [13], Zhao and Tang [22], Chen et al. [2], Ji et al. [11], Luo [12], and Zhang [21]. Except for Ji et al. [10], none of the abovementioned due window assignment research has taken grouping or batching decisions of jobs into consideration.

Yin et al. [18,19] have studied a common due window assignment and scheduling problem with batching decisions, where the sizes of the delivery batches are unconstrained. The challenge in solving this problem is that the objective function comprises six different cost components, namely the earliness penalty of jobs, the tardiness penalty of jobs, the cost associated with the start time of the due window, the cost associated with the size of the due window, the holding cost of jobs while they are waiting for delivery, and the cost of each delivery. The study by Yin et al. [18] considered a cost function in which the tardiness penalty is the total job tardiness, while the study by Yin et al. [19] considered a cost function in which the tardiness penalty is the weighted number of tardy jobs. In this paper, we consider three different variants of the problem studied by Yin et al. [18,19], and we present improved algorithms with significantly lower computational complexities than those obtained by them.

The rest of the paper is organized as follows. In Section 2, we provide a mathematical description of our problem and the three variants that we study. In Sections 3, 4, and 5, the improved algorithms for these three variants are presented. Some concluding remarks are made in Section 6.

2. Problem definitions and preliminaries

Mathematically, our due window assignment and scheduling problem can be described as follows (see Yin et al. [18,19]): We are given n independent jobs $J_1, J_2, \ldots, J_n$ to be processed on a single machine, where J_j has a processing time p_j ($j = 1, 2, \ldots, n$). All jobs are available for processing at time zero, and job preemption is not allowed. The finished jobs are to be delivered to the customer in batches, and the delivery time of a batch equals the completion time of the last job in the batch. In other words, the jobs of a batch are processed consecutively. Once the processing of one batch is completed, the batch is delivered to the customer immediately, and the machine will continue to process the jobs in the next batch. There is no capacity limit on each batch. The cost per delivery is fixed; that is, the cost is independent of the number of jobs delivered in a batch. All n jobs have a common due window, and the start time and size of the due window are decision variables. There is a (job-independent) earliness cost if a job is delivered earlier than the due window, and there is a tardiness penalty if a job is delivered later than the due window.

Let d_1 and d_2 be the start time and finish time of the due window, respectively, where $d_1 \le d_2$. Let C_j and D_j be the completion time and delivery time of J_j, respectively, where $D_j \ge C_j$. A job is said to be early if $D_j \le d_1$, and tardy if $D_j > d_2$. Let $E_j = \max\{d_1 - D_j, 0\}$ and $T_j = \max\{D_j - d_2, 0\}$ be the earliness and tardiness, respectively, of J_j. Let $H_j = D_j - C_j$ be the holding time of J_j, which is the duration between the moment that J_j finishes processing till the moment that J_j is delivered. Let $U_j = 1$ if $D_j > d_2$, and $U_j = 0$ otherwise. Thus, U_j indicates whether J_j is tardy or not. Let $D = d_2 - d_1$ be the size of the due window, and m be the total number of delivery batches. Let α be the unit cost of earliness, θ be the holding cost per job per unit time, γ be the cost of delaying the start time of the due window, δ be the cost of increasing the size of the due window, and ψ be the delivery cost of each batch. We consider three different variants of the due-window assignment problem, namely $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$, which correspond to three different measurements of tardiness penalty. The total cost of these three variants are given as

$$Z_1 = \sum_{j=1}^{n} (\alpha E_j + \beta_j U_j + \theta H_j + \gamma d_1 + \delta D) + m\psi,$$

$$Z_2 = \sum_{j=1}^{n} (\alpha E_j + \beta U_j + \theta H_j + \gamma d_1 + \delta D) + m\psi,$$