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Bionomics dynamic model of a class of competition systems ☆

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ABSTRACT

Differential equation problem is an important research topic in the international academia. In accordance with certain ecological phenomena, previous research was conducted based on simple observational and statistical data. But this approach does not effectively study the essence of the ecological phenomena. Recently, one dynamic approach has been proposed for the study of ecology in the international academia. According to this approach, first of all, the ecology is reduced to the differential equation model which represents the essential phenomenon, and then the dynamic law and rules of mathematics and biology will be studied. Currently, an extensive research is conducted on the differential equation problem. This paper primarily explores a type of competitive ecological model, which is a system of differential equation with infinite integral. we first study the existence of positive periodic solution to this model, and then present sufficient conditions for the global attractivity of positive periodic solutions.

1. Introduction

In recent years, there has been a rapid development among the application of theories of differential equations in mathematical ecology. Tremendous research effort has been spent on the dynamics of population which is useful for the control of the environment and population of human beings and animals. One of the most celebrated models is the competition system. For instance, Kuang [1] proposed the following delayed three species competition system

$$\begin{cases} x_1'(t)=x_1(t)\left[1-x_1(t)-\int_{-\infty}^{t}K(s-t)x_2(s)ds-\int_{-\infty}^{t}L(s-t)x_3(s)ds\right] \\ x_2'(t)=x_2(t)\left[1-\int_{-\infty}^{t}L(s-t)x_1(s)ds-x_2(t)-\int_{-\infty}^{t}K(s-t)x_3(s)ds\right] \\ x_3'(t)=x_3(t)\left[1-\int_{-\infty}^{t}K(s-t)x_1(s)ds-\int_{-\infty}^{t}L(s-t)x_2(s)ds-x_3(t)\right], \end{cases}$$

and only obtained the permanent coexistence. And other competition models have been also studied in [2–10].

In this paper, we investigate the following delayed three species competition system.

$$\begin{cases} x_1'(t)=x_1(t)\left[r_1(t)-x_1(t)-\int_{-\infty}^{t}K(s-t)x_2(s)ds-\int_{-\infty}^{t}L(s-t)x_3(s)ds\right] \\ x_2'(t)=x_2(t)\left[r_2(t)-\int_{-\infty}^{t}L(s-t)x_1(s)ds-x_2(t)-\int_{-\infty}^{t}K(s-t)x_3(s)ds\right] \\ x_3'(t)=x_3(t)\left[r_3(t)-\int_{-\infty}^{t}K(s-t)x_1(s)ds-\int_{-\infty}^{t}L(s-t)x_2(s)ds-x_3(t)\right], \end{cases} \tag{1.1}$$

where $x_i(t)(i=1,2,3)$ denotes the density of competing species at time t, $r_i \in C(R,[0,+\infty))$ is ω-periodic function ($\omega > 0$), K and L are nonnegative functions in $L_1(-\infty,0]$ with

$$\bar{r}_i = \frac{1}{\omega}\int_0^{\omega} r_i(s)ds > 0, \quad i=1,2,3, \tag{1.2}$$

$$k_1 = \int_{-\infty}^{0} K(\theta)d\theta > 0, \quad k = \frac{1}{\omega}\int_0^{\omega} dt\int_{-\infty}^{t} K(s-t)ds > 0, \tag{1.3}$$

$$l_1 = \int_{-\infty}^{0} L(\theta)d\theta > 0, \quad l = \frac{1}{\omega}\int_0^{\omega} dt\int_{-\infty}^{t} L(s-t)ds > 0. \tag{1.4}$$

The main purpose of this paper is to study the periodic solutions of system (1.1). More precisely, in Section 2, we study the existence of positive periodic solutions by using Krasonoselskii's fixed point theorem. In Section 3, we present sufficient conditions for the global attractivity of positive periodic solutions by constructing a Lyapunov function.

Throughout of this paper, we say a vector $x=(x_1,x_2,x_3)^T$ is positive if $x_i > 0(i=1,2,3)$.

2. The existence of positive periodic solutions

Lemma 2.1 (*Krasnoselskii 11*). *Let X be a Banach space, and let $P \subset X$ be a cone in X. Assume that Ω_1, Ω_2 are open bounded subsets of X with $0 \in \Omega_1 \subset \overline{\Omega}_1 \subset \Omega_2$, and let*

$$\varphi : P \cap (\overline{\Omega}_2 \setminus \Omega_1) \to P,$$

be a completely continuous operator such that either:

(i) $\|\varphi x\| \leqslant \|x\|$, $\forall x \in P \cap \partial\Omega_1$ *and* $\|\varphi x\| \geqslant \|x\|$, $\forall x \in P \cap \partial\Omega_2$;
(ii) $\|\varphi x\| \geqslant \|x\|$, $\forall x \in P \cap \partial\Omega_1$ *and* $\|\varphi x\| \leqslant \|x\|$, $\forall x \in P \cap \partial\Omega_2$.

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Then φ has a fixed point in $P \cap (\bar{\Omega}_2 \setminus \Omega_1)$.

Let

$$X = \left\{x(t) = (x_1(t), x_2(t), x_3(t))^T \in C(R, R^3) : x(t+\omega) = x(t)\right\}. \quad (2.1)$$

$$\|x\| = \sum_{j=1}^{3} |x_j|_0, \quad |x_j|_0 = \max_{t\in[0,\omega]} |x_j(t)|, \quad j = 1,2,3. \quad (2.2)$$

Then X is a Banach space endowed with the above norm $\|\cdot\|$. If $x(t) = (x_1(t), x_2(t), x_3(t))^T \in X$ is a solution of system (1.1), then

$$\begin{cases} \left[x_1(t)\exp\left(-\int_0^t r_1(s)ds\right)\right]' = -\exp\left(-\int_0^t r_1(s)ds\right)x_1(t)\left[x_1(t) + \int_{-\infty}^t K(s-t)x_2(s)ds + \int_{-\infty}^t L(s-t)x_3(s)ds\right] \\ \left[x_2(t)\exp\left(-\int_0^t r_2(s)ds\right)\right]' = -\exp\left(-\int_0^t r_2(s)ds\right)x_2(t)\left[\int_{-\infty}^t L(s-t)x_1(s)ds + x_2(t) + \int_{-\infty}^t K(s-t)x_3(s)ds\right] \\ \left[x_3(t)\exp\left(-\int_0^t r_3(s)ds\right)\right]' = -\exp\left(-\int_0^t r_3(s)ds\right)x_3(t)\left[\int_{-\infty}^t K(s-t)x_1(s)ds + \int_{-\infty}^t L(s-t)x_2(s)ds + x_3(t)\right]. \end{cases} \quad (2.3)$$

Integrating both sides of (2.3) over $[t, t+\omega]$, we obtain

$$\begin{cases} x_1(t) = \int_t^{t+\omega} G_1(t,\sigma)x_1(\sigma)\left[x_1(\sigma) + \int_{-\infty}^{\sigma} K(s-\sigma)x_2(s)ds + \int_{-\infty}^{\sigma} L(s-\sigma)x_3(s)ds\right]d\sigma \\ x_2(t) = \int_t^{t+\omega} G_2(t,\sigma)x_2(\sigma)\left[\int_{-\infty}^{\sigma} L(s-\sigma)x_1(s)ds + x_2(\sigma) + \int_{-\infty}^{\sigma} K(s-\sigma)x_3(s)ds\right]d\sigma \\ x_3(t) = \int_t^{t+\omega} G_3(t,\sigma)x_3(\sigma)\left[\int_{-\infty}^{\sigma} K(s-\sigma)x_1(s)ds + \int_{-\infty}^{\sigma} L(s-\sigma)x_2(s)ds + x_3(\sigma)\right]d\sigma, \end{cases} \quad (2.4)$$

where

$$G_i(t,\sigma) = \frac{1}{1-e^{-\bar{r}_i\omega}} \exp\left(-\int_t^{\sigma} r_i(s)ds\right), \quad i = 1,2,3. \quad (2.5)$$

Let $\eta = \min\{e^{-\bar{r}_i\omega} : i = 1,2,3\}$. Now, choose a cone defined by

$$P = \left\{x(t) = (x_1(t), x_2(t), x_3(t))^T \in X : x_i(t) \geqslant \eta |x_i|_0, \quad i = 1,2,3\right\}, \quad (2.6)$$

and define an operator $\Phi: X \to X$ by

$$(\Phi x)(t) = ((\Phi x)_1(t), (\Phi x)_2(t), (\Phi x)_3(t))^T, \quad (2.7)$$

where

$$\begin{cases} (\Phi x)_1(t) = \int_t^{t+\omega} G_1(t,\sigma)x_1(\sigma)\left[x_1(\sigma) + \int_{-\infty}^{\sigma} K(s-\sigma)x_2(s)ds + \int_{-\infty}^{\sigma} L(s-\sigma)x_3(s)ds\right]d\sigma \\ (\Phi x)_2(t) = \int_t^{t+\omega} G_2(t,\sigma)x_2(\sigma)\left[\int_{-\infty}^{\sigma} L(s-\sigma)x_1(s)ds + x_2(\sigma) + \int_{-\infty}^{\sigma} K(s-\sigma)x_3(s)ds\right]d\sigma \\ (\Phi x)_3(t) = \int_t^{t+\omega} G_3(t,\sigma)x_3(\sigma)\left[\int_{-\infty}^{\sigma} K(s-\sigma)x_1(s)ds + \int_{-\infty}^{\sigma} L(s-\sigma)x_2(s)ds + x_3(\sigma)\right]d\sigma. \end{cases} \quad (2.8)$$

Through the simple calculation, we can obtain that $(\Phi x)_i(t+\omega) = (\Phi x)_i(t)$, $(i = 1,2,3)$, so we have $(\Phi x)(t+\omega) = (\Phi x)(t)$. Then by (2.4) and (2.8), we have that $x^*(t) = (x_1^*(t), x_2^*(t), x_3^*(t))^T \in X$ is a positive ω – periodic solution of system (1.1) provided that $x^*(t)$ is a fixed point of Φ.

Lemma 2.2. *The mapping Φ maps P into P, i.e. $\Phi P \subset P$.*

Proof. It is easy to see that for $t \leqslant \sigma \leqslant t+\omega$.

$$A_i := \frac{e^{-\bar{r}_i\omega}}{1-e^{-\bar{r}_i\omega}} \leqslant G_i(t,\sigma) \leqslant \frac{1}{1-e^{-\bar{r}_i\omega}} =: B_i, \quad i = 1,2,3. \quad (2.9)$$

From (2.8) to (2.9), we have for $x \in P$,

$$|(\Phi x)_1|_0 \leqslant B_1 \int_0^{\omega} x_1(\sigma)\left[x_1(\sigma) + \int_{-\infty}^{\sigma} K(s-\sigma)x_2(s)ds + \int_{-\infty}^{\sigma} L(s-\sigma)x_3(s)ds\right]d\sigma,$$

and

$$(\Phi x)_1 \geqslant A_1 \int_0^{\omega} x_1(\sigma)\left[x_1(\sigma) + \int_{-\infty}^{\sigma} K(s-\sigma)x_2(s)ds + \int_{-\infty}^{\sigma} L(s-\sigma)x_3(s)ds\right]d\sigma \geqslant \frac{A_1}{B_1}|(\Phi x)_1|_0 \geqslant \eta|(\Phi x)_1|_0.$$

Similarly, we can obtain that $(\Phi x)_i \geqslant \eta|(\Phi x)_i|_0$, $(i = 2,3)$. Therefore, $\Phi P \subset P$, the proof is completed. □

Lemma 2.3. *$\Phi: P \to P$ is completely continuous.*

Proof. Let

$$\begin{cases} f_1(t,x_t) = x_1(t)\left[x_1(t) + \int_{-\infty}^t K(s-t)x_2(s)ds + \int_{-\infty}^t L(s-t)x_3(s)ds\right] \\ f_2(t,x_t) = x_2(t)\left[\int_{-\infty}^t L(s-t)x_1(s)ds + x_2(t) + \int_{-\infty}^t K(s-t)x_3(s)ds\right] \\ f_3(t,x_t) = x_3(t)\left[\int_{-\infty}^t K(s-t)x_1(s)ds + \int_{-\infty}^t L(s-t)x_2(s)ds + x_3(t)\right]. \end{cases}$$

We first show that Φ is continuous. For any $L > 0$ and $\varepsilon > 0$, there exists a $\delta > 0$ such that for any $x, y \in X$, $\|x\| \leqslant L$, $\|y\| \leqslant L$, and $\|x-y\| < \delta$ imply

$$\max_{t\in[0,\omega]} |f_i(t,x_t) - f_i(t,y_t)| < \frac{\varepsilon}{3B\omega} \quad i = 1,2,3, \quad (2.10)$$

where $B = \max_{1\leqslant i\leqslant 3} B_i$, then from (2.8) to (2.10), we have

$$|(\Phi x)_i - (\Phi y)_i|_0 \leqslant B\int_0^{\omega} |f_i(\sigma,x_\sigma) - f_i(\sigma,y_\sigma)|d\sigma < \frac{\varepsilon}{3}, \quad i = 1,2,3.$$

This yields

$$\|\Phi x - \Phi y\| = \sum_{i=1}^{3} |(\Phi x)_i - (\Phi y)_i|_0 < \varepsilon.$$

Thus, Φ is continuous.

Next, we prove that Φ is a compact operator. Let $M > 0$ be any constant and let $S = \{x \in X: \|x\| \leqslant M\}$ be a bounded set. For any $x \in S$, it follows from (2.8), (2.9), (1.3) and (1.4) that

$$|(\Phi x)_1|_0 \leqslant BM \times \int_0^{\omega}\left[M + M\int_{-\infty}^{\sigma} K(s-\sigma)ds + M\int_{-\infty}^{\sigma} L(s-\sigma)ds\right]d\sigma = BM^2\omega(1+k+l). \quad (2.11)$$

Similarly, we can obtain that

$$|(\Phi x)_i|_0 \leqslant BM^2\omega(1+k+l), \quad i = 2,3, \quad (2.12)$$

and so

$$\|\Phi x\| = \sum_{j=1}^{3} |(\Phi x)_j|_0 \leqslant 3BM^2\omega(1+k+l), \quad \forall x \in S.$$

Furthermore, from (2.8), we have

$$\begin{cases} (\Phi x)_1'(t) = r_1(t)(\Phi x)_1(t) - x_1(t)\left[x_1(t) + \int_{-\infty}^t K(s-t)x_2(s)ds + \int_{-\infty}^t L(s-t)x_3(s)ds\right] \\ (\Phi x)_2'(t) = r_2(t)(\Phi x)_2(t) - x_2(t)\left[\int_{-\infty}^t L(s-t)x_1(s)ds + x_2(t) + \int_{-\infty}^t K(s-t)x_3(s)ds\right] \\ (\Phi x)_3'(t) = r_3(t)(\Phi x)_3(t) - x_3(t)\left[\int_{-\infty}^t K(s-t)x_1(s)ds + \int_{-\infty}^t L(s-t)x_2(s)ds + x_3(t)\right]. \end{cases}$$