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Around probabilistic cellular automata

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ABSTRACT

We survey probabilistic cellular automata with approaches coming from combinatorics, statistical physics, and theoretical computer science, each bringing a different viewpoint. Some of the questions studied are specific to a domain, and some others are shared, most notably the ergodicity problem.

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1. Introduction

Consider a set of cells indexed by $\mathbb{Z}^d$, $d \geq 1$, each cell containing a letter from a finite alphabet $\mathcal{A}$. The updating of cells is local (each cell updates according to a finite neighborhood), time-synchronous, and space-homogeneous. When the updating is deterministic, we obtain a Cellular Automaton (CA), and when it is random, we obtain a Probabilistic Cellular Automaton (PCA). Alternatively, PCA may be viewed as discrete-time Markov chains on the state space $\mathcal{A}^{\mathbb{Z}^d}$ which are the synchronous counterparts of (finite range) interacting particle systems.

CA are important and widely studied for at least three reasons. First, they are natural from a dynamical point of view. In particular, by Hedlund's theorem [37], they correspond precisely to the functions on $\mathcal{A}^{\mathbb{Z}^d}$ which are continuous (with respect to the product topology) and commuting with translations. Second, they constitute a powerful model of computation, in particular they can "simulate" any Turing machine. Third, due to the amazing gap between the simplicity of their definition and the intricacy of their trajectories, CA are good candidates for modeling "complex systems" appearing in physical and biological processes.

This multiplicity of viewpoints carries over to PCA. First, in theoretical computer science, PCA obtained as a random perturbation of CA have been considered, with at least two different motivations: – to investigate the fault-tolerant computation capability of CA; – to classify elementary CA (Wolfram's program) by using the robustness to errors as a discriminating criterion. Second, in statistical mechanics, general PCA are studied for their connections with Gibbs potentials and Gibbs measures. Third, some specific PCA are linked to important combinatorial models, in particular, directed animals, queues, and directed percolation. Last, PCA inherit from CA the ability to be relevant models for complex systems appearing in physics and biology.

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There exist several books and surveys dedicated to CA, see [43] and the references therein. For PCA, the standard reference is the textbook [75] from 1990. The more recent survey articles [72,74] mostly deal with fault-tolerant computations. In the present survey, our primary goal was to keep the balance between the different viewpoints and to present them in a unified way. We focus on some aspects for each domain, rather than seeking to be exhaustive.

2. Preliminaries on probabilistic cellular automata

2.1. Informal definition of PCA and some examples

Consider an infinite lattice, e.g. $\mathbb{Z}$ or $\mathbb{Z}^2$, divided in regular cells, each cell containing a letter of a finite alphabet. At each time step, the content of a cell changes randomly according to a probability law which depends on the content of a finite neighborhood of the cell. The updates of the different cells are done independently. This is the rough definition of a *Probabilistic Cellular Automaton (PCA)*. In a nutshell, the dynamics is local, random, time-synchronous, and space-homogeneous.

A *Cellular Automaton (CA)* is a degenerated PCA in which, for each neighborhood, the update probability law is concentrated on a single letter, that is, the updates are deterministic.

Consider the specific case of the set of cells $\mathbb{Z}$, the alphabet $\{0, 1\}$, and the neighborhood consisting of the cell itself and its right neighbor (or, the left neighbor and the cell itself). Then, a PCA is entirely determined by the four parameters $(\theta_{00}, \theta_{01}, \theta_{10}, \theta_{11})$, where $\theta_{ij} \in [0, 1]$ is the probability that a cell is updated to 1 if its neighborhood is ij. Here are some examples.

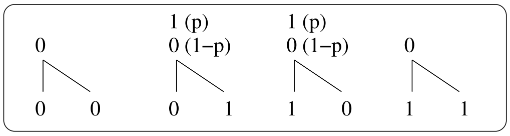

Fig. 1. The noisy additive PCA.

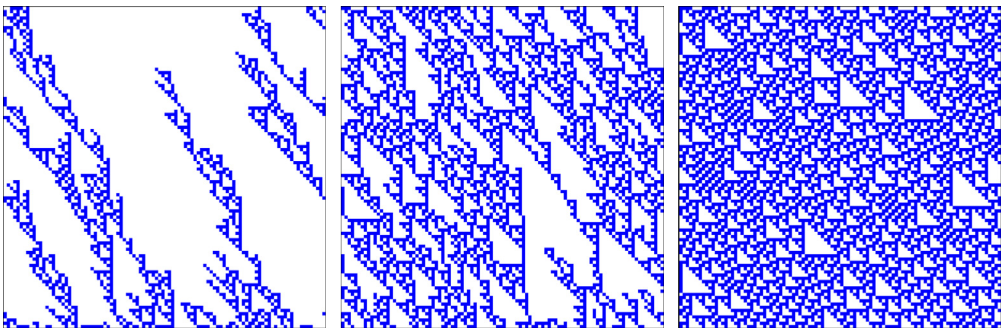

Fig. 2. Space–time diagrams of the noisy additive PCA, for $p = 0.75$, $p = 0.85$, and $p = 1$. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Noisy additive PCA Consider the PCA defined by the parameters $(0, p, p, 0)$ for some $p \in (0, 1)$, see Fig. 1.

This PCA can be described as follows. Let the neighborhood of a cell be $(i, j) \in \{0, 1\}^2$, then, the cell is updated in two steps: first, its value is set to $i + j \bmod 2$, and second, with probability $1 - p$, a value 1 is turned into 0. In the limit case $1 - p = 0$, we recover the *additive CA* also known as rule 102 in Wolfram's notation and defined by:

$$F : \{0, 1\}^{\mathbb{Z}} \longrightarrow \{0, 1\}^{\mathbb{Z}}, \qquad x \longmapsto F(x), \qquad F(x)_i = x_i + x_{i+1} \bmod 2. \tag{1}$$

When $p \in (0, 1)$, we get a PCA which can be viewed as a "noisy" version of the additive CA.

In Fig. 2, we have represented the evolution of the noisy additive PCA for different values of the parameter p in so-called *space–time diagrams*. The cells containing a 0, resp. a 1, are painted in white, resp. dark blue. The bottom line is the initial condition, here chosen at random, and the next lines, from bottom to top, are the successive updates of the cells.

One may consider a symmetric variant of the noisy additive PCA: the model in Fig. 3 of parameters $(1 - p, p, p, 1 - p)$ for some $p \in (0, 1)$. Here the cell is updated in two steps: first, its value is set to $i + j \bmod 2$, and second, with probability $1 - p$, a value 1 is turned into 0 and a value 0 is turned into 1.

Some space–time diagrams are shown in Fig. 4. They are qualitatively very different from the ones in Fig. 2.

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