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Non-planar square-orthogonal drawing with few-bend edges

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ABSTRACT

We investigate square-orthogonal drawings of non-planar graphs with vertices represented as unit grid squares. We present quadratic-time algorithms to construct the square-orthogonal drawings of 5-graphs, 6-graphs, and 8-graphs such that each edge in the drawing contains at most two, two, and three bends, respectively. In particular, the novel analysis method we use to split a vertex so as to build some specific propagation channels in our algorithms is an interesting technique and may be of independent interest. Moreover, we show that the decision problem of determining whether an 8-graph has a square-orthogonal drawing without edge-bends is NP-complete.

1. Introduction

A *drawing* (or *embedding*) of a graph in the plane roughly consists of a representation of the vertices by objects, an assignment of the vertices to geometric positions in the plane, and a representation of the edges by simple open curves between the objects representing two vertices. In particular, in an *orthogonal drawing*, edges are represented by simple polygonal chains consisting of horizontal and vertical segments. Forcing a graph to be drawn orthogonally has the advantage of showing the maximal distinctiveness of adjacent edges in the drawing. The price we may need to pay for such type of clarity is to admit bends in the path representing an edge. In order to avoid confusion caused by too complex paths, it is desirable to place a low number of bends on each edge.

A *k-graph* is a graph of maximum vertex degree k. For an orthogonal drawing under the model that vertices are drawn as grid points (see Fig. 1(a)), Tamassia [11] showed that the planar drawing with minimum number of bends on edges for a plane 4-graph can be computed in near quadratic time. However, any plane k-graph with $k \geqslant 5$ does not have orthogonal drawing. In order to draw graphs with higher vertex degree, and with simple shapes representing vertices, we investigate the orthogonal drawing with vertices represented as unit grid squares, which is called *square-orthogonal drawing* (see Fig. 1(b)). Researchers also considered even more general objects like rectangles to represent the vertices of the graph, for instance [2–6,9]. However, one clear disadvantage of using rectangular shapes to represent vertices is that rectangles could be fat or skinny, and hence irregular. This sometimes will hinder the readability of the whole drawing of a graph. We observe that orthogonal drawing with consistent shapes for all vertices may have the advantage of clarity for visualization. Some researchers [2,4,5] also investigated the planar orthogonal drawing under the model of representing vertices by uniformly small squares. However, in their drawings, adjacent edges are allowed to run in parallel with very small gap between them, which hinders the readability and clarity of some specific parts of the drawing. Instead, in our representation model, only edges lying on the grid lines are allowed, which prevents adjacent edges from congesting together. We focus on drawing non-planar graphs whereas some of the motivating work above only applies to planar graphs. Moreover, with our

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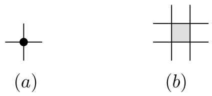

Fig. 1. Vertex drawn as (i) a grid point; or (ii) a unit grid square.

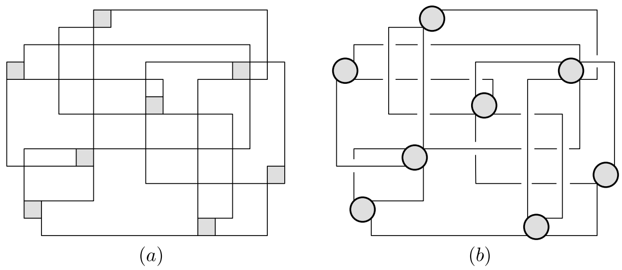

Fig. 2. (a) Vertices drawn as unit grid squares. (b) Vertices drawn as unit-diameter disks, and edges cased.

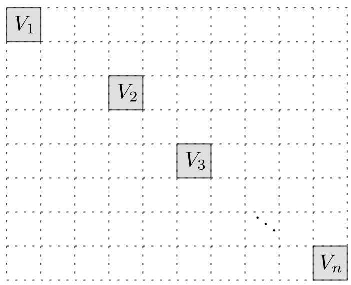

Fig. 3. The vertices are embedded along a diagonal.

current representation model, we can afford to draw k-graphs, where k can be at most 8. Note that in our drawings, edge crossings are allowed, but edge overlapping is not allowed (see Fig. 2(a)). In our original formulation, nodes are represented as unit grid squares, and edges can cross each other; however, in practice, nodes can in fact be represented as unit-diameter disks, and the intersecting edges can be drawn using some well-known techniques, such as edge-casing (see Fig. 2(b)).

Previous work Let n and m be the number of vertices and edges in the input graph. We first consider the research work under the model of representing vertices by points on the plane. Schaffter [10] presented an algorithm to compute an orthogonal drawing, for 4-graphs, of $2n \times 2n$ grid and with at most two bends on each edge. Biedl and Kant [1] gave a linear-time algorithm to compute an orthogonal drawing of $n \times n$ grid area and with at most $2n + 2$ total number of edge bends. Papakostas and Tollis [7,8] further improved the area upper bound to $0.76n^2$ grid area while keeping the same bound on total number of edge bends. For graphs with vertex degrees higher than four, Biedl, Madden and Tollis [3] used the model of representing vertices by rectangles, and they presented an algorithm to compute an orthogonal drawing on $(\frac{m+n}{2}) \times (\frac{m+n}{2})$ grid and with at most one bend per edge. Later, Papakostas and Tollis [9] improved the drawing area to be within $(m-1) \times (\frac{m}{2} + 2)$ while keeping the same bound on edge bends.

The rest of this paper is organized as follows. In Section 2, we introduce some definitions and terminologies. In Sections 3 and 4, we present algorithms to construct the square-orthogonal drawing for 5-graphs and 6-graphs such that each edge contains at most two bends, and to construct the square-orthogonal drawing of 8-graphs such that each edge contains at most three bends. In Section 5, we show that the decision problem of determining whether an 8-graph has a square-orthogonal drawing without edge-bends is NP-complete.

2. Preliminaries

The framework of our algorithms is as follows. At the very beginning of the algorithm, we embed the vertices $V = \{v_1, v_2, \dots, v_n\}$ of the input graph G diagonally, beginning at the top-left corner and towards the bottom-right corner, reserving for each vertex v_i a unit square V_i (see Fig. 3). We remark that the horizontal and vertical distances between two adjacent vertices will be determined after all edges in the graph are drawn. We then proceed to insert the edges of G one by one. Each time we insert an edge, if we cannot establish an immediate connection with low bend number, we will make some modifications to the related edges so that the target bend number is attained for the inserted edge. Below we introduce some conventions and definitions.