Theoretical Computer Science 554 (2014) 263–274

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Social exchange networks with distant bargaining ☆

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ARTICLE INFO

Available online 1 December 2013

Keywords:
Bargaining
Exchange networks
Cooperative games

ABSTRACT

Network bargaining is a natural extension of the classical, 2-player *Nash bargaining* solution to the network setting. Here one is given an exchange network G connecting a set of players V in which edges correspond to potential contracts between their endpoints. In the standard model, a player may engage in at most one contract, and feasible outcomes therefore correspond to matchings in the underlying graph. Kleinberg and Tardos [STOC'08] recently proposed this model, and introduced the concepts of *stability* and *balance* for feasible outcomes. The authors characterized the class of instances that admit such solutions, and presented a polynomial-time algorithm to compute them.

In this paper, we generalize the work of Kleinberg and Tardos by allowing agents to engage into more complex contracts that span more than two agents. We provide suitable generalizations of the above stability and balance notions, and show that many of the previously known results for the matching case extend to our new setting. In particular, we can show that a given instance admits a stable outcome only if it also admits a balanced one. Like Bateni et al. [ICALP'10] we exploit connections to cooperative games. We fully characterize the core of these games, and show that checking its non-emptiness is NP-complete. On the other hand, we provide efficient algorithms to compute core elements for several special cases of the problem, making use of compact linear programming formulations.

1. Introduction

The study of bargaining has been a central theme in economics and sociology, since it constitutes a basic activity in any human society. The most basic bargaining model is that of two agents A and B that negotiate how to divide a good of a certain value (say, 1) amongst themselves, while at the same time each has an *outside option* of value α and β respectively. The famous Nash bargaining solution [1] postulates that in an *equitable* outcome, each player should receive her outside option, and that the surplus $s = 1 - \alpha - \beta$ is to be split evenly between A and B.

More recently, Kleinberg and Tardos [2] proposed the following natural *network* extension of this game. Here, the set of players corresponds to the vertices of an undirected graph $G = (V, E)$; each edge $ij \in E$ represents a potential *contract*

☆ This work was initiated in the International Problem Solving Workshop, held in July 16–20, 2012 and organized as part of the MITACS International Focus Period "Advances in Network Analysis and its Applications". We would like to thank MITACS for this great opportunity. Research partially supported by NSERC.

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0304-3975/$ – see front matter
http://dx.doi.org/10.1016/j.tcs.2013.11.033

between players i and j of value $w_{ij} \geqslant 0$. In Kleinberg and Tardos' model, players are restricted to form contracts with at most one of their neighbours. Outcomes of the *network bargaining* game are therefore given by a matching $M \subseteq E$, and an *allocation* $x \in \mathbb{R}^V_+$ such that $x_i + x_j = w_{ij}$ for all $ij \in M$, and $x_i = 0$ if i is not incident to an edge of M.

Unlike in the non-network bargaining game, the outside option α_i of player is not a given parameter but rather implicitly determined by the network neighbourhood of i. Specifically, in an outcome (M, x), player i's outside option is defined as $\alpha_i = \max\{w_{ij} - x_j\colon\ ij \in \delta(i) \setminus M\}$, where $\delta(i)$ is the set of edges incident to i. An outcome (M, x) is then called *stable* if $x_i + x_j \geqslant w_{ij}$ for all edges $ij \in E$, and it is *balanced* if in addition, the value of the edges in M is split according to Nash's bargaining solution; i.e., for an edge ij, $x_i - \alpha_i = x_j - \alpha_j$. Kleinberg and Tardos provide a characterization of the class of graphs that admit balanced outcomes, and present a combinatorial algorithm that computes one if it exists.

Bateni et al. [3] recently exhibited a close link between the study of network bargaining and that of *matching games* in cooperative game theory. The authors showed that stable outcomes for an instance of network bargaining correspond to allocations in the *core* of the underlying matching game. Moreover, balanced outcomes correspond to *prekernel* allocations. As a corollary, this implies that an algorithm by Faigle et al. [4] gives an alternate method to obtain balanced outcomes in a network bargaining game. Bateni et al. also extended the work of [2] to bipartite graphs in which the agents of one side are allowed to engage in more than one contract.

Matching games have indeed been studied extensively in the game theory community since the early 70s, when Shapley and Shubik investigated the core of the class of bipartite matching games, so-called *assignment games*, in their seminal paper [5]. Granot and Granot [6] also study the core of the assignment game; the authors show that it contains many points, some of which may not be desirable ways to share revenue. The authors propose to focus on the intersection of core and prekernel instead, and provide sufficient and necessary conditions for the former to be contained in the latter. Deng et al. [7] generalized the work of Shapley and Shubik to matchings in general graphs as well as to cooperative games of many other combinatorial optimization problems. We refer the reader also to the recent survey paper [8] and the excellent textbook [9].

In this paper we further generalize the work of [2] and [3] on network bargaining by allowing contracts to span more than two agents. Our study is motivated by bargaining settings where goods are complex composites of other goods that are under the control of autonomous agents. For example, in a computer network setting, two hosts A and B may wish to establish a connection between themselves. Any such connection may involve physical links from a number of smaller autonomous networks that are provisioned by individual players. In this setting, value generated by the connection between A and B cannot merely be shared by the two hosts, but must also be used to compensate those *facilitators* whose networks the connection uses.

1.1. Generalized network bargaining

We formalize the above ideas by defining the class of *generalized network bargaining* (GNB) games. In an instance of such a game, we are given a (directed or undirected) graph $G = (V, E)$ whose vertices correspond to players, and edges that correspond to atomic goods; the *value* of the good corresponding to e is given by $w_e \geqslant 0$. We assume that V is partitioned into *terminals* T, and *facilitators* R. Intuitively, the terminals are the *active* players that seek participation in contracts, while facilitators are *passive*, and may get involved in contracts, but do not seek involvement. We further let $\mathcal{C}$ be a family of *contracts* each of whom consists of a collection of atomic goods. We let $w(c)$ be the *value* of contract c which we simply define as the sum of values w_e of the edges $e \in c$. We note here that in the work of [2] and [3], $\mathcal{C}$ consists just of the singleton edges.

A set $C \subseteq \mathcal{C}$ of contracts is called *feasible* if each two contracts in C are vertex disjoint. An *outcome* of an instance of GNB is given by a feasible collection $C \subseteq \mathcal{C}$ as well as an allocation $x \in \mathbb{R}^V_+$ of the contract values to the players such that

$$x(c) := \sum_{v \in c} x_v = w(c).$$

Which outcomes are *desirable*? We propose the following natural extensions of the notions of *stability* and *balance* of [2]. Consider an outcome (C, x) of some instance of GNB. Then define the *outside option* α_i of player i as

$$\alpha_i := \max_{c \in \mathcal{C}\colon\ i \in c \notin C} \bigl\{ w(c) - x(c) \bigr\} + x_i.$$

Intuitively, the outside option of i is given by the value she can earn by breaking her current contract, and participating in a contract that is not part of the current outcome. We will assume that each agent i is incident to a self-loop of value 0, and hence has the option of not collaborating with anyone else. In what follows $a(c) := \sum_{v \in c} a_v$ for a contract $c \in C$.

Having defined α_i, we can now introduce the notions of *stability* and *balance*. An outcome (C, x) is stable if $x_i \geqslant \alpha_i$ for all agents i: every agent earns at least her outside option. Again extending the concept of Nash bargaining solution in the most natural way, we say that an outcome is balanced if the surplus of each contract is shared evenly among the participating agents. Formally, for all $c \in C$, and for all $i \in c$ we require

$$x_i = \alpha_i + \frac{w(c) - \alpha(c)}{|c|}.$$

Equivalently, this means that $x_i - \alpha_i = x_j - \alpha_j$ for all $i, j \in c$, and for all $c \in C$.