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The fashion game: Network extension of Matching Pennies

Zhigang Cao, Xiaoguang Yang *

Key Laboratory of Management, Decision & Information Systems, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, 100190, PR China

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ABSTRACT

It is impossible, in general, to extend an asymmetric two-player game to networks, because there must be two populations, the row one and the column one, but we do not know how to define inner-population interactions. This is not the case for Matching Pennies, as we can interpret the row player as a conformist, who prefers to coordinate her opponent's action, while the column player can be interpreted as a rebel, who likes to anti-coordinate. Therefore we can naturally define the interaction between two conformists as the coordination game, and that between two rebels as the anti-coordination game. It turns out that the above network extension of Matching Pennies can be used to investigate the phenomenon of fashion, and thus it is named the fashion game. The fashion game possesses an obvious mixed Nash equilibrium, yet we are especially interested in pure Nash equilibrium (PNE for short), whose existence cannot be guaranteed. In this paper, we focus on the PNE testing problem, namely given an instance of the fashion game, answer whether it possesses a PNE or not. Our first result is on the negative side: PNE testing, in general, is hard. For the PNE testing problem restricted to several special structures, i.e. lines, rings, complete graphs and trees, either a simple characterization or an efficient algorithm is provided.

"Fashion is the most intense expression of the phenomenon of neomania, which has grown ever since the birth of capitalism."

[Stephen Bayley]

"Fashion has been one of the most influential phenomena in Western civilization since the Renaissance. It has conquered an increasing number of modern man's fields of activity and has become almost 'second nature' to us."

[Lars Svendsen]

1. Introduction

Network games, also known as graphical games, games on graphs and games on networks, roughly speaking, are games that are played on networks. Here the network represents the social connections between players, and payoffs of players are only affected by their neighbors. But action sets of players are usually independent of this social network (notice this difference with routing games on networks). Therefore, any strategic game can be taken as a network game, because we

* Corresponding author.
E-mail addresses: zhigangcao@amss.ac.cn (Z. Cao), xgyang@iss.ac.cn (X. Yang).

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can simply let the associated network be complete. Sitting on the intersection of algorithmic game theory, social economics, social physics, social networks (as well as complex networks), and theoretical biology, the field of network games is typically multi-disciplinary. We refer the reader to [16–19,21] for more detailed introduction to this booming field.

A wide class of network games are those that are generalized from two player strategic form games. Theoretically, any two player symmetric game (referred to as the base game) can be generalized to networks in the following way. Each player, denoted by a node of the network, plays the base game once with each of her neighbors. The total payoff of each player is simply defined as the sum of her payoff obtained in all the base games that she is involved. Also, it is usually assumed that for any player, in all the base games that she plays, she should take the same action.

Using a similar logic on an asymmetric two-player game, there must be two populations introduced: the row population and the column population. When two players from different populations meet, they play the original two-player game. Nothing is debatable for this. However, complication comes when two players from the same population meet: we don't know, in general, how to define the game they play. This information is generally not written in the asymmetric two-player game. This is exactly why scholars from evolutionary game theory in the very little literature simply ignore the inner-population game when they deal with asymmetric two-player games. For instance, if the row population stands for deer and column for lions, they only study the relation between deer and lions, but not that among deer or among lions.

In this paper, we shall investigate a network game that is generalized from Matching Pennies, an elementary model in game theory that can be found in any textbook. Note first that Matching Pennies is not symmetric. It turns out that this model is closely related with the phenomenon of fashion.

1.1. The fashion game

Fashion is a very interesting phenomenon that plays a critical role not only in economy but also in many other areas of our society (see [11,23] for more detailed discussions).

On what is fashionable, interestingly, there are two viewpoints that are both extremely popular but almost opposite to each other. One point of view thinks that fashion is a distinctive or peculiar manner or way. Lady Gaga is regarded as fashionable in this sense. The other takes fashion as a prevailing custom or style. Fashion color is fashionable in this latter sense. This difference has a very deep root in psychology (see [11] for more discussions). Following Young [22] and Jackson [18], we shall call the former type of people *rebels*, and the latter *conformists*.

Since fashion comes from comparison with others, and the range that people compare is almost always confined to their friends, relatives, colleagues, and neighbors, that fashion works through a social network is the most natural thing. As far as we know, the model we study is first formulated by Jackson (2008, [18], p. 271), where it is presented as an example of graphical games. It is also studied in Young (2001, [22], p. 38), but the social network is not explicitly expressed there.

Formally, each fashion game is represented by a triple $G = (N, E, T)$, where $N = \{1, 2, \cdots, n\}$ is the set of nodes that stand for agents (we may also interchangeably use the term *players*), $E \subseteq N \times N$ the set of edges (no self-loops are allowed), and $T \in \{C, R\}^N$ the configuration of types. For agent $i \in N$, T_i is her type: $T_i = C$ means that i is a conformist, and $T_i = R$ means that she is a rebel. For agents $i, j \in N$, they are neighboring to each other if and only if $ij \in E$. If $ij \in E$, then $ji \in E$, i.e. the network is undirected. The action sets of all agents are identical: $\{0, 1\}$.

We use N_i to denote the set of neighbors of player i, and $\pi_i \in \{0, 1\}$ the action of agent i. Vector $\pi \in \{0, 1\}^N$ is an action profile of G. Given an action profile π, for all $i \in N$, $L_i(\pi) \subseteq N_i$ is the set of neighboring agents that i likes, i.e.

$$L_i(\pi) = \begin{cases} \{j \in N_i : \pi_j = \pi_i\} & \text{if } T_i = C \\ \{j \in N_i : \pi_j \neq \pi_i\} & \text{if } T_i = R \end{cases}$$

Similarly, $D_i(\pi) \subseteq N_i$ is the set of neighboring agents that i dislikes, i.e. $D_i(\pi) = N_i \setminus L_i(\pi)$. Consequently, the utility of each player i is defined as the number of neighbors she likes minus that she dislikes, i.e.

$$u_i(\pi) = \left|L_i(\pi)\right| - \left|D_i(\pi)\right|.$$

In the rest of this paper, $u_i(\pi)$ may also be referred to as the *satisfaction degree*. An agent is called *satisfied* if $u_i(\pi) \geq 0$. Otherwise, she is called *unsatisfied*. Straightforwardly, an action profile π is a Nash equilibrium if and only if every agent is satisfied.

We may also abuse the term of graph (and network) by calling an instance of the fashion game a graph. For example, when we say that $G = (N, E, T)$ is a tree, we actually mean that the underlying graph (N, E) is a tree. That is, the type configuration T may be simply overlooked.

1.2. Network extension of Matching Pennies

It turns out that, as we have mentioned, the fashion game may also be taken as the network extension of Matching Pennies. First of all, observe that Matching Pennies is equivalent to a very special fashion game: there are two players in total, one is a conformist and the other a rebel, who correspond to the row player and the column player, respectively, and the underlying network is a dyad.