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Bisimulations for weighted automata over an additively idempotent semiring ☆

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ABSTRACT

We show that the methodology for deciding the existence and computation of the greatest simulations and bisimulations, developed in the framework of fuzzy automata (Ćirić et al., 2012) [13,14], can be applied in a similar form to weighted automata over an additively idempotent semiring. We define two types of simulations and four types of bisimulations for weighted automata over an additively idempotent semiring as solutions to particular systems of matrix inequalities, and we provide polynomial-time algorithms for deciding whether there is a simulation or bisimulation of a given type between two weighted automata, and for computing the greatest one, if it exists. The algorithms are based on the concept of relative Boolean residuation for matrices over an additively idempotent semiring, that we introduce here. We also prove that two weighted automata $\mathscr{A}$ and $\mathscr{B}$ are forward bisimulation equivalent, i.e., there is a row and column complete forward bisimulation between them, if and only if the factor weighted automata with respect to the greatest forward bisimulation equivalence matrices on $\mathscr{A}$ and $\mathscr{B}$ are isomorphic. In addition, we show that the factor weighted automaton with respect to the greatest forward bisimulation equivalence matrix on a weighted automaton $\mathscr{A}$ is the unique (up to an isomorphism) minimal automaton in the class of all weighted automata which are forward bisimulation equivalent to $\mathscr{A}$.

1. Introduction

Simulation and bisimulation relations are very powerful tools that have been used in many areas of computer science to match moves and compare the behavior of various systems, as well as to reduce the number of states of these systems. They have been introduced by Milner [26] and Park [28] in computer science, i.e., in concurrency theory, but roughly at the same time they have been also discovered in some areas of mathematics, e.g., in modal logic and set theory. Afterwards, the use of simulation and bisimulation has gained a long and rich history and their various forms have been defined and applied to different systems. They are employed today in the study of functional languages, object-oriented languages, types, data types, domains, databases, compiler optimizations, program analysis, verification tools, etc.

The most common structures on which simulations and bisimulations have been studied are labeled transition systems, but they have also been investigated in the context of deterministic, nondeterministic, weighted, fuzzy, probabilistic, timed,

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hybrid and other kinds of automata. One type of bisimulation for weighted automata has been introduced by Ésik and Kuich [17] (and for Boolean automata by Bloom and Ésik [7]), under the name simulation. Under the same name this concept has been studied by Ésik and Maletti [18], and under different names in [3–5,9,25,29]. Béal and Perrin [5] used the name backward elementary equivalence, Béal, Lombardy, and Sakarovitch [3,4] the name conjugacy, which originates from applications in symbolic dynamics, and Buchholz [9] used the name (forward) relational simulation. It is important to note that Béal, Lombardy, and Sakarovitch [3,4] found that a semiring S often has the following property: two weighted automata over S are equivalent if and only if they are connected by a finite chain of simulations. Semirings having this property include the Boolean semiring [7], the semiring of natural numbers and the ring of integers [3,4], etc. An example of a semiring which does not have this property is the tropical semiring [18]. In addition, the mentioned concept was used in [17] in the completeness proof of the iteration semiring theory axioms for regular languages. Note also that Ésik and Kuich's simulations, as well as Béal, Lombardy, and Sakarovitch's conjugacies, were defined as matrices over a semiring which satisfy the same conditions which we use here for Boolean matrices to define backward–forward bisimulations, one of our four types of bisimulations. Buchholz's definition uses the same conditions, but his bisimulations are defined to be exclusively relational or functional matrices.

A new approach to simulations and bisimulations has been recently proposed in [13,14], in the framework of fuzzy automata, and in [11] it has been applied to ordinary nondeterministic automata. Two types of simulations and four types of bisimulations for fuzzy automata have been defined as fuzzy relations (fuzzy matrices) which are solutions to certain systems of fuzzy relation (matrix) inequalities. Such an approach cannot be directly applied to weighted automata over an arbitrary semiring, because it requires an ordering of matrices, and therefore, an ordering in the underlying semiring, and, in general, the underlying semiring does not have to be ordered. On the other hand, even if the underlying semiring is ordered, we have the problem how to check the solvability and find solutions to the corresponding systems of matrix inequalities and compute simulations and bisimulations. Algorithms developed in [14], for testing the existence of simulations and bisimulations between fuzzy automata and computing the greatest ones (when they exist), are strongly based on the concept of residuation for fuzzy matrices, which is an immediate consequence of residuation in the underlying structures of truth values. However, residuation is present only in some special classes of semirings, such as, for example, max-plus and related semirings. The main aim of this paper is to show that these problems can be partially solved when the weights are taken from an additively idempotent semiring. Such semirings possess a natural ordering which allows us to define simulations and bisimulations by means of systems of matrix inequalities. Moreover, the zero and unit of an additively idempotent semiring form a subsemiring isomorphic to the Boolean semiring, and consequently, matrices with entries in this subsemiring can be treated as Boolean matrices, that is, as ordinary binary relations. Finally, although in general there is no residuation for matrices over an additively idempotent semiring, there is some kind of relative residuation that results in a Boolean matrix. This enables us to find solutions of our systems of matrix inequalities in the class of Boolean matrices, and therefore, to develop the theory of simulations and bisimulations based on relational matrices (ordinary binary relations).

Our main results are the following. We define two types of simulations and four types of bisimulations for weighted automata over an additively idempotent semiring, but because of the duality we further consider only one type of simulations and two types of bisimulations. They are defined to be relational matrices. We also introduce the concepts of relative right and left residuals for matrices over an additively idempotent semiring, and prove the corresponding adjunction properties (Theorems 5.1 and 5.2). Using relative residuals we provide an equivalent form of conditions that define forward simulations (Theorem 5.3), which enables us to give a polynomial-time algorithm for deciding whether there is a forward simulation between two weighted automata, and whenever there is at least one such simulation, the same algorithm computes the greatest one (cf. Theorem 5.4 and Algorithm 5.5). We also point out how to construct the corresponding algorithms for forward bisimulations and backward–forward bisimulations. Special attention is paid to bisimulations that are uniform Boolean matrices. We show that a uniform Boolean matrix is a forward bisimulation if and only if its kernel and co-kernel are forward bisimulation equivalence matrices and there is a special isomorphism between related factor weighted automata (Theorem 6.1). A similar theorem for backward–forward bisimulations is also given (Theorem 6.8). Furthermore, we prove that if two weighted automata $\mathscr{A}$ and $\mathscr{B}$ are forward bisimulation equivalent, i.e., if there is a uniform forward bisimulation between them, then there is the greatest forward bisimulation between $\mathscr{A}$ and $\mathscr{B}$ and it is a uniform Boolean matrix, whose kernel and co-kernel are respectively the greatest forward bisimulation equivalence matrices on $\mathscr{A}$ and $\mathscr{B}$ (cf. Theorem 6.5). Then we get that two weighted automata $\mathscr{A}$ and $\mathscr{B}$ are forward bisimulation equivalent if and only if the factor weighted automata with respect to the greatest forward bisimulation equivalence matrices on $\mathscr{A}$ and $\mathscr{B}$ are isomorphic. Also, we show that the factor weighted automaton of an arbitrary weighted automaton $\mathscr{A}$ with respect to the greatest forward bisimulation equivalence matrix on $\mathscr{A}$ is the unique (up to an isomorphism) minimal automaton in the class of all weighted automata that are forward bisimulation equivalent to $\mathscr{A}$ (Theorem 6.7). Finally, we prove that the concepts of a forward and a backward–forward bisimulation coincide when working with functional matrices (cf. Theorem 6.9).

The structure of the paper is as follows. In Section 2 we introduce basic notions and notation related to additively idempotent semirings and matrices over them, as well as to Boolean matrices. Special attention is paid to uniform Boolean matrices. Then in Section 3 we give basic notions and notation concerning weighted automata. Section 4 contains our definitions of simulations and bisimulations, and results describing their basic properties. In Section 5 we provide algorithms for checking the existence and computing the greatest simulations and bisimulations. At the end, Section 6 contains results that characterize uniform forward and backward–forward bisimulations, and forward bisimulation equivalent weighted