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Monoid-matrix type automata

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ABSTRACT

Monoid-matrix type automata are introduced and studied in this paper. We give a characterization of the cyclic monoid-matrix type automata and the regular monoid-matrix type automata. Also, we provide a method to determine the structures of canonical $S\ell$-automata (canonical C-automata, respectively) whose endomorphism monoids are isomorphic to a given finite meet semilattice with the greatest element (Clifford monoid, respectively).

1. Introduction

An automaton $\mathbf{A} = (A, \Sigma, \delta)$ consists of the following data:

(i) A is a finite nonempty set of states;
(ii) Σ is a finite nonempty set, called an alphabet;
(iii) $\delta : A \times \Sigma \to A$, called a state transition function.

Let Σ^* denote the *free monoid* generated by Σ. An element of Σ^* is called a *word* over Σ and ϵ is called the *empty word*. The state transition function can be extended to the function from $A \times \Sigma^*$ to A as follows:

(i) $(\forall a \in A)\ \delta(a, \epsilon) = a$;
(ii) $(\forall a \in A, x \in \Sigma, u \in \Sigma^*)\ \delta(a, ux) = \delta(\delta(a, u), x)$.

Let $\mathbf{A} = (A, \Sigma, \delta)$ and $\mathbf{B} = (B, \Sigma, \gamma)$ be automata. A mapping f from A into B is called *a homomorphism from* $\mathbf{A}$ *into* $\mathbf{B}$ if $f(\delta(a, x)) = \gamma(f(a), x)$ holds for any $a \in A$ and $x \in \Sigma$. If a homomorphism f is bijective, then f is called *an isomorphism*. If there exists an isomorphism from $\mathbf{A}$ onto $\mathbf{B}$, then $\mathbf{A}$ and $\mathbf{B}$ are said to be *isomorphic* to each other and denoted by $\mathbf{A} \cong \mathbf{B}$. Moreover, a homomorphism (an isomorphism) from $\mathbf{A}$ into itself is called *an endomorphism* (*an automorphism*) *of* $\mathbf{A}$. It is clear that $E(\mathbf{A})(G(\mathbf{A}))$ of all endomorphisms (automorphisms) of $\mathbf{A}$ forms a monoid (group) on the usual composition, called the *endomorphism monoid* (*automorphism group*) *of* $\mathbf{A}$.

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An automaton $\mathbf{A}$ is called *G-automaton* (*Sℓ-automaton*, *C-automaton*, respectively), if $E(\mathbf{A})$ is a group (meet semilattice, Clifford monoid, respectively).

The study of automorphism groups and endomorphism monoids of automata was initiated by Fleck (1962, 1965) and Weeg (1962) in [1–4], and followed by Barners (1965, 1970), Bayer (1966), Trauth (1966), Shibata (1972), Masunaga et al. (1973), and Dörfler (1978) in [5–11].

Let $\mathbf{A} = (A, \Sigma, \delta)$ be an automaton. If $\delta(s, aa) = \delta(s, a)$ holds for any $s \in A$ and any $a \in \Sigma$, then $\mathbf{A}$ is said to be asynchronous. A state a in A is called a *generator* of $\mathbf{A}$ [10] if for any $b \in A$, there exists $u \in \Sigma^*$ such that $\delta(a, u) = b$. The set of all generators of $\mathbf{A}$ is denoted by $Gen(\mathbf{A})$, called the *generating set* of $\mathbf{A}$. An automaton $\mathbf{A}$ is said to be *cyclic* (*strongly connected*) if $Gen(\mathbf{A}) \neq \emptyset$ ($Gen(\mathbf{A}) = A$). Obviously, if $\mathbf{A}$ is a strongly connected automaton, then for any pair of states $a, b \in A$, there exists a word $u \in \Sigma^*$ such that $\delta(a, u) = b$, and the reverse is also true.

Based on the study of the automorphism group of a strongly connected automaton, Ito [12–17] provided a representation of a strongly connected automaton $\mathbf{A} = (A, \Sigma, \delta)$ by regular group-matrix type automaton $\mathbf{A}' = (\widehat{G(\mathbf{A})_n}, \Sigma, \delta_\Psi)$ of order n on the automorphism group $G(\mathbf{A})$ such that $\mathbf{A}' \cong \mathbf{A}$. Further, given a finite group G, he gave a method to determine the structures of strongly connected automata whose automorphism groups are isomorphic to G.

Following Ito, Tian and Zhao introduced the notion of canonical automaton in [18,19]. They proved both strongly connected automaton and cyclic commutative asynchronous automaton are canonical. Also, they provided the representation of canonical automata by regular monoid-matrix type automata. This generalizes and extends Ito's result on the representation of strongly connected automata.

Developing Ito's idea, we study the structures of canonical $S\ell$-automata and canonical C-automata by means of regular monoid-matrix type automata.

In Section 2, some notions and notations, such as canonical automata and monoid-matrix type automata, are recalled. In Section 3, the cyclic monoid-matrix type automata and the regular monoid-matrix type automata are studied and their characterizations are given, respectively. Further, in Section 4, we give a necessary and sufficient condition for two regular (n, S)-automata isomorphic to each other in the wider sense. Based on these results, we provide in Section 5 (Section 6, respectively) a method to determine the structures of canonical $S\ell$-automata (canonical C-automata, respectively) whose endomorphism monoids are isomorphic to a given finite meet semilattice with the greatest element (Clifford monoid, respectively).

2. Preliminaries

Recall the following notions and notations which will be of use in the later.

Let $\mathbf{A} = (A, \Sigma, \delta)$ be a cyclic automaton. A binary relation $\mathcal{L}$ on $\mathbf{A}$ is defined as follows:

$$\mathcal{L} \triangleq \{(a, b) \in A \times A \mid (\exists s \in Gen(\mathbf{A}))\ a, b \in O_s\},$$

where O_s denotes the set $\{f(s) | f \in E(\mathbf{A})\}$. If $\mathcal{L}$ is an equivalence relation on A, then $\mathbf{A}$ is said to be *canonical* in [19]. A canonical automaton is said to be canonical G-automaton (canonical $S\ell$-automaton, canonical C-automaton, respectively), if it is a G-automaton ($S\ell$-automaton, C-automaton, respectively). Tian [19] provided a representation of canonical automata by the regular monoid-matrix type automata.

Suppose that $(S, \cdot)$ is a finite monoid with the identity 1_S. If we adjoin an extra element 0 to the set S and define

$$0 \cdot 0 = 0 \quad \text{and} \quad s0 = 0s = s \quad \text{for any } s \in S,$$

then $S \cup \{0\}$ becomes a semigroup with the zero element 0, which is denoted by ${}^0S$. Also, we define a partial operation $+$ on $S \cup \{0\}$ as follows:

* $0 + 0 = 0$ and $s + 0 = 0 + s = s$ for any $s \in S$;
* $s + s'$ has no sense for any $s, s' \in S$.

Let n be a positive integer and $\alpha = (a_1, a_2, \ldots, a_n)$ a row vector over ${}^0S$. If there exists a unique positive number $i \in \{1, 2, \ldots, n\}$ such that $a_i \neq 0$, then α is called *a monoid-vector of order n on S* [19]. We shall denote the set of all monoid-vectors of order n on S by $\widehat{S_n}$. Define $s\alpha$ as the usual scalar product for any $s \in S$ and any $\alpha \in \widehat{S_n}$.

Put

$$\varepsilon_1 = (1_S, 0, \ldots, 0)$$
$$\varepsilon_2 = (0, 1_S, \ldots, 0)$$
$$\cdots$$
$$\varepsilon_n = (0, 0, \ldots, 1_S).$$

Then $\widehat{S_n} = \{s\varepsilon_i \mid s \in S,\ i = 1, 2, \ldots, n\}$.