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Reachability problems in BioAmbients

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ABSTRACT

BioAmbients (BA) is a powerful model for representing various aspects of living cells. The model provides a rich set of operations for the movement and interaction of molecules. The richness of the language motivates the study of fragments of the full model and comparison with other computational models. In this paper, we investigate the impact of the *merge* capability, used for fusing the contents of two sibling ambients, on the decidability of two reachability problems called Target and Spatial Reachability. By enhancing techniques – based on the theory of Petri nets – already used in the context of Mobile Ambients, we prove that both Target and Spatial Reachability are decidable for a Turing-complete fragment of BA without *merge*. Then we extend this fragment with a limited form of *merge*, that does not reduce the total number of ambients: in this fragment Target Reachability is no longer decidable, but by resorting to the theory of Petri nets with transfer arcs we prove that at least Spatial Reachability is decidable. Finally, we show that if we consider the standard *merge* capability then both reachability problems turn out to be undecidable. Besides characterizing the power of *merge*, the proof techniques that we use also establish an interesting connection between BA and other computational models like standard Petri nets, their extension with transfer arcs, and Two Counter Machines.

1. Introduction

BioAmbients [17] (BA) is a well known formalism for the description of biological systems that combines the communication mechanisms of the π-calculus [16] with the notion of ambient as formalized in Mobile Ambients [7]. This combination allows for the representation of biochemical reactions by means of process communication and to model biological compartments by means of ambients. A BioAmbient $[P]$ is a collection of active processes and nested sub-BioAmbients represented by the term P. Active processes can perform communication actions with other processes or execute capabilities in order to modify the ambient nesting. Communication consists of the interaction between an output and an input action performed by processes located in the same ambient, in parent/child ambients, or in two parallel ambients. The capabilities allow processes to modify ambient nesting in three possible ways: one ambient can move inside a parallel ambient, one ambient can move outside from the parent ambient, or two parallel ambients can merge into one single ambient.

In this paper, we discuss reachability problems in BA by exploiting and enhancing techniques developed by Busi and Zavattaro for Mobile and Boxed Ambients [4–6]. Classical reachability analysis consists in verifying, given a source process P and a target process Q, whether there exists a computation that starts from P and leads to Q. For ambient-based calculi, extensions of the reachability problem have been considered due to the presence of the ambient nesting structure. In [5,6]

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the Target Reachability problem has been defined: instead of considering a single target process Q, this problem allows for the specification of a possibly infinite class of targets. The set of the specifiable targets includes processes all having a given ambient nesting structure, and such that each of those ambients satisfy some given specific constraints. The constraints allow for the specification of lower and/or upper bounds to the number of instances of some given sequential processes (intuitively, a sequential process is a process which is not the parallel composition of other subprocesses and it is not an ambient containing other subprocesses). A simpler version of this problem, called Spatial Reachability, was introduced in [4] where only lower bounds could be expressed.

As an example of the kind of analysis allowed by Target and Spatial Reachability problems that we propose, let us consider a generic modeling of the cell represented as an outer membrane that contains the intermediate cytoplasm and the inner nucleus. We could then consider two different external materials, one to which the membrane should be permeable, and another one to which it should not be.

Such a system could be modeled in BA in the following way

$$[Material1] \mid [Material2] \mid [Membrane \mid [Cytoplasm \mid [Nucleus]]]$$

where the first two ambients are used to represent and describe the behavior of two different kinds of external material, and the other ambient represents the cell with its outer membrane, the intermediate cytoplasm, and the inner nucleus.

We can formalize the expected behavior of the system in terms of reachability problems. For instance, to formalize the fact that the membrane should be permeable for the first kind of material we can state that we expect that the configuration

$$[Material2] \mid [Membrane' \mid [[Material1'] \mid Cytoplasm' \mid [Nucleus]]]$$

is reachable, where $Membrane'$, $Material1'$ and $Cytoplasm'$ respectively describe the expected state of the membrane, of the transported material, and of the cytoplasm after the membrane has been traversed. This kind of reachability property, i.e., reachability of a given process, is expressible in terms of Target Reachability by imposing the lower bounds for the processes to be present equal to their upper bounds.

On the contrary, to formalize that the membrane should not be permeable for the second kind of material, we can state that we expect that the configuration

$$[Material1] \mid [Membrane'' \mid [[Material2''] \mid Cytoplasm'' \mid [Nucleus]]]$$

is not reachable for any process $Membrane''$, $Material2''$ and $Cytoplasm''$. This different kind of reachability property is already expressible in terms of Spatial Reachability by indicating only the expected ambient nesting structure, and imposing no constraints on the contained processes.

Reachability is usually undecidable in Turing complete formalisms such as the π-calculus or Mobile Ambients (the ancestors of BA). Nevertheless, at least for Mobile Ambients, very interesting fragments have been studied which are expressive enough to model all computable functions, but for which reachability problems turn out to be decidable.

Charatonik and Talbot proved in [8] the undecidability of reachability in the fragment of Mobile Ambients without name restriction. The undecidability result was enforced by Boneva and Talbot [3] who proved that reachability is undecidable even if the capability to dissolve ambient boundaries is removed. The proof of undecidability makes use of the possibility to apply the replication operator $!P$ also to ambients in order to represent an unbounded number of replica of an ambient P. By applying the usual congruence rule $!P \equiv !P|P$ from right to left, it is possible to remove from a term an active ambient P. In the same paper Boneva and Talbot showed that if the congruence rule $!P \equiv !P|P$ is replaced by a reduction rule $!P \rightarrow !P|P$ then reachability turns out to be decidable: intuitively, this follows from the fact that the number of active ambients cannot decrease. Another fragment in which the number of active ambients cannot decrease has been studied by Maffeis and Phillips: instead of changing the congruence rules, this fragment is obtained by allowing the application of replication only to processes and not to ambients. This fragment was proved to be Turing complete in [15]. Reachability problems were subsequently studied for this fragment by Busi and Zavattaro, and both Spatial and Target Reachability were proved to be decidable [4–6].

In this paper we apply and extend these results and techniques to BA. We start by considering a fragment of BA similar to the one considered in [15,4]. By resorting to the results in [15] we first show that this fragment is Turing complete, then we show that Target Reachability is decidable by adapting to this new context the techniques in [6] based on the theory of Petri nets. Then we consider an extension of this fragment that includes a limited version of the *merge* capability: every time two ambients are merged, then at least a new ambient is created. In this way, the "monotonicity" property about the number of active ambients is preserved. Interestingly, we prove that although monotonicity is preserved Target Reachability is no longer decidable, while the simpler Spatial Reachability problem still is decidable. This result is proved by resorting to the theory of Petri nets with transfer arcs. Finally, we show that if we lose the monotonicity property by admitting a *merge* mechanism that can also decrease the total number of active ambients, then also Spatial Reachability becomes undecidable.

Structure of the paper

In Section 2 we report the syntax and semantics of BA^-, the fragment of BioAmbients that we obtain by removing the choice operator, the communication primitives, the restriction operator, and by imposing that replication is applied only to processes and not to ambients. The elimination of the choice operator and communication primitives is done only for