Available online at www.sciencedirect.com
ScienceDirect
Theoretical Computer Science 379 (2007) 306–316
Theoretical Computer Science
www.elsevier.com/locate/tcs

Degree distribution of the FKP network model

Noam Berger[a], Béla Bollobás[b], Christian Borgs[c], Jennifer Chayes[c], Oliver Riordan[d,*]

[a] Department of Statistics, University of California, Berkeley, CA 94720, United States
[b] Department of Mathematical Sciences, University of Memphis, Memphis TN 38152, United States
[c] Microsoft Research, One Microsoft Way, Redmond, WA 98122, United States
[d] Department of Pure Mathematics, University of Cambridge, Cambridge CB3 0WB, UK

Abstract

Power laws, in particular power-law degree distributions, have been observed in real-world networks in a very wide range of contexts, including social networks, biological networks, and artificial networks such as the physical internet or abstract world wide web. Recently, these observations have triggered much work attempting to explain the power laws in terms of new 'scale-free' random graph models. So far, perhaps the most effective mechanism for explaining power laws is the combination of growth and preferential attachment. In [A. Fabrikant, E. Koutsoupias, C.H. Papadimitriou, Heuristically optimized trade-offs: A new paradigm for power laws in the internet ICALP 2002, in: LNCS, vol. 2380, pp. 110–122], Fabrikant, Koutsoupias and Papadimitriou propose a new 'paradigm' for explaining power laws, based on trade-offs between competing objectives. They also introduce a new, simple and elegant parametrized model for the internet, and prove some kind of power-law bound on the degree sequence for a wide range of scalings of the trade-off parameter.

Here we shall show that this model does *not* have the usual kind of power-law degree distribution observed in the real world: for the most interesting range of the parameter, neither the bulk of the nodes, nor the few highest degree nodes have degrees following a power law. We shall show that almost all nodes have degree 1, and that there is a strong bunching of degrees near the maximum.
© 2007 Elsevier B.V. All rights reserved.

Keywords: Power-law degree distribution; HOT model

1. Introduction

Recently there has been an explosion of interest in the mathematical study of large-scale real-world networks, in particular, in the development of simple random graph models to explain certain observed common features. Perhaps the most striking such feature, seen in a very wide range of contexts, is that the networks are 'scale-free': the distribution of degrees in the graph follows a power law, as do distributions of many other characteristics. Although such power laws were known in various contexts many decades ago (see, for example, [10,15,16]), recent work perhaps started from the observations of Faloutsos, Faloutsos and Faloutsos [8] on the 'internet graph', the graph (at a suitable level) of the physical connections forming the internet.

* Corresponding author.
E-mail address: O.M.Riordan@dpmms.cam.ac.uk (O. Riordan).

0304-3975/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.tcs.2007.02.049

These observations have led to a host of proposals for 'scale-free' random graph models to explain these power laws, and to better understand the mechanisms at work in the growth of real-world networks such as the internet or web graphs; see [2,3,9] for a few examples. For extensive surveys of the huge amount of work in this area, see Albert and Barabási [1] and Dorogovtsev and Mendes [6]; for a survey of the rather smaller quantity of mathematical work see [4].

Most of the models introduced use a small number of basic mechanisms to produce power laws. So far, the most successful is the combination of growth in time with some form of 'preferential attachment' or 'rich get richer' mechanism, which may arise indirectly via copying, for example. Examples are the vague Barabási-Albert model [2] or the precise LCD model [3], as well as the copying model of [9], and many others. Such models tend not to be realistic in any one context; the idea is to suggest a simple mechanism at work (together with other factors, of course) in many different contexts, which may be responsible for the prevalence of power laws.

Taking the particular example of the internet graph, there is a very good reason for expecting the models mentioned above not to fit very well: these models operate entirely on the abstract graph, ignoring any differences between nodes, or pre-existing structure on the set of nodes. When deciding how to wire a physical network, location, and in particular physical distances between nodes, will be very important.

In [7], Fabrikant, Koutsoupias and Papadimitriou (FKP) proposed a new paradigm for power-law behaviour, which they called 'heuristically optimized trade-offs': power laws may result from 'complicated optimization problems with multiple and conflicting objectives'. Their paradigm generalizes previous work by Carlson and Doyle [5] on 'highly optimized tolerance', in which reliable design is one of the objectives. More specifically, FKP introduced a specific very simple, natural and elegant new model for the growth of certain networks, in particular the internet graph, involving a trade-off between network and geometric distances. They suggest that the degree distribution of this model follows a power law. However, we shall show here that any power law that is followed must have a very unusual form, differing from the forms actually observed in many contexts, and in particular for the internet graph [8].

Thus, while the FKP model is interesting and the mechanism introduced will be important in many contexts, there is little evidence that this model, or rather the general idea of 'heuristically optimized trade-offs', provides a new paradigm for power laws as suggested in [7].

We now turn to the specific model introduced by Fabrikant, Koutsoupias and Papadimitriou. As in many models, the network is grown one node at a time, and each node chooses a previous node to which it connects. However, in contrast to other network models, a key feature of the FKP model is the underlying geometry; the nodes are points chosen uniformly at random from some region, for example a unit square in the plane. The trade-off is between the geometric consideration that it is desirable to connect to a nearby point, and a networking consideration, that it is desirable to connect to a node which is 'central' in the network as a graph. Centrality may be measured by using, for example, the graph distance to the initial node.

Several variants of the basic model are considered by Fabrikant, Koutsoupias and Papadimitriou in [7]. The precise version we shall consider here is the principal version studied in [7]: fix a region $\mathcal{D}$ of area one in the plane, for example a disk or a unit square. The model is then determined by the number of nodes, $n + 1$, and a parameter, α. We start with a point x_0 of $\mathcal{D}$ chosen uniformly at random, and set $W(x_0) = 0$. For $i = 1, 2, \ldots, n$ we choose a new point x_i of $\mathcal{D}$ uniformly at random, and connect x_i to an earlier point x_j chosen to minimize

$$W(x_j) + \alpha d(x_i, x_j)$$

over $0 \le j < i$. Here $d(.,.)$ is the usual Euclidean distance. Having chosen x_j, we set $W(x_i) = W(x_j) + 1$. At the end, we have a random tree $T = T(n, \alpha)$ on $n + 1$ nodes $x_0, \ldots, x_n$, where each node has a weight $W(x_i)$ which is just its graph distance in the tree from x_0.

As usual, for mathematical results we are most interested in the asymptotic behaviour of the model as $n \to \infty$. The parameter α will be a function of n, typically a constant power.

One might think from the title or a first reading of [7] that the form of the degree sequence of this model has been essentially established. In fact, as we shall describe in the next section, this is not the case. Indeed, two of our results, while of course consistent with the actual results of [7], go against the impression given there that the entire degree sequence follows a power law.

The work described in this paper was first presented at ICALP 2003, and appears in a shorter form in the proceedings.

Download English Version:

https://daneshyari.com/en/article/439305

Download Persian Version:

https://daneshyari.com/article/439305

Daneshyari.com