

## Hierarchical grid conversion



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### ABSTRACT

Hierarchical grids appear in various applications in computer graphics such as subdivision and multiresolution surfaces, and terrain models. Since the different grid types perform better at different tasks, it is desired to switch between regular grids to take advantages of these grids. Based on a 2D domain obtained from the connectivity information of a mesh, we can define simple conversions to switch between regular grids. In this paper, we introduce a general framework that can be used to convert a given grid to another and we discuss the properties of these refinements such as their transformations. This framework is hierarchical meaning that it provides conversions between meshes at different level of refinement. To describe the use of this framework, we define new regular and near-regular refinements with good properties such as small factors. We also describe how grid conversion enables us to use patch-based data structures for hexagonal cells and near-regular refinements. To do so, meshes are converted to a set of quadrilateral patches that can be stored in simple structures. Near-regular refinements are also supported by defining two sets of neighborhood vectors that connect a vertex to its neighbors and are useful to address connectivity queries.

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### 1. Introduction

Triangular, quadrilateral, and hexagonal grids appear in many applications in computer graphics such as finite elements, subdivision and multiresolution surfaces, and terrain rendering. Triangular grids are common due to their application in many fundamental algorithms such as Delaunay triangulation and Loop subdivision [1,2], and they are also optimized for processing on modern hardware. The simple parametric form of quadrilateral grids can be readily applied to tensor product surfaces, NURBS, B-Spline, and Catmull–Clark patches [3,4]. Furthermore, quadtrees [5] exploit the simple boundaries of quadrilateral grids and their straightforward hierarchical shape. Hexagonal grids provide the best sampling of surfaces as they provide less bias towards edges (they are more circular) in comparison with squares and triangles, support uniform neighborhood, and provide a reduced quantization error over other alternatives [6]. As a result, hexagonal grids appear in applications such as hierarchical representation of the Earth and subdivision surfaces [7,8].

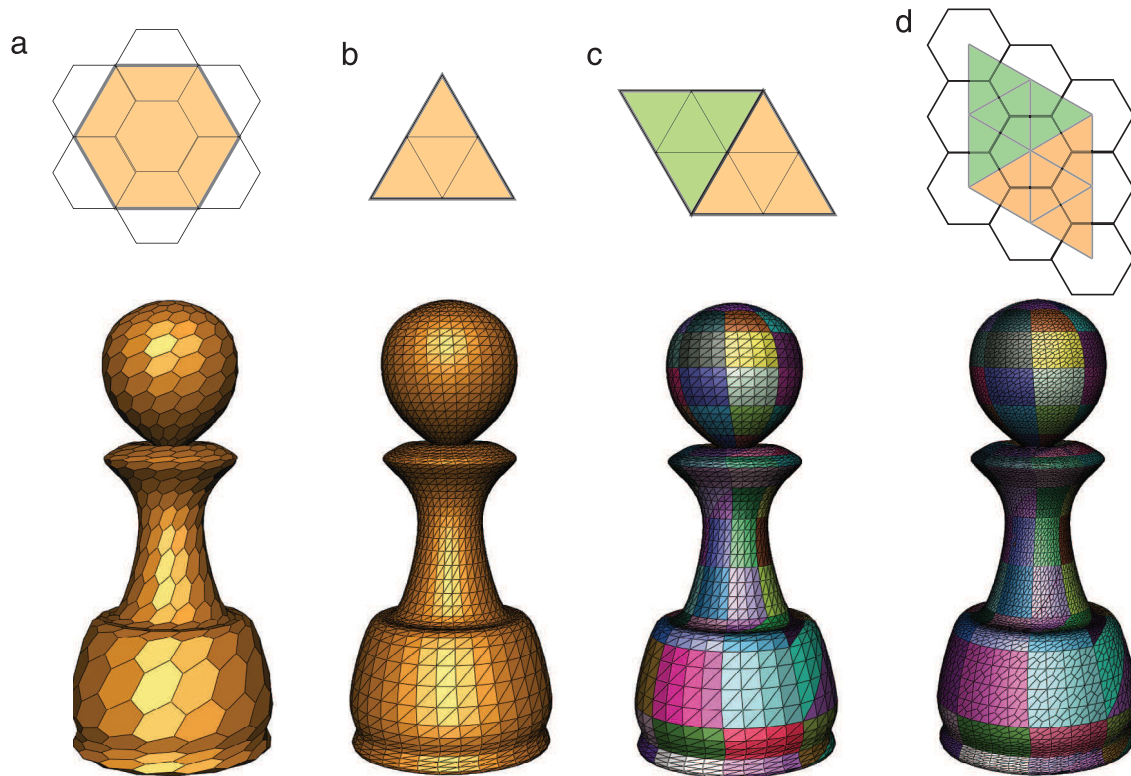
In this paper, we provide hierarchical grid conversions between triangular, quadrilateral and hexagonal grids. These conversions

are basically simple modifications in the connectivity of vertices that convert a type of grid to another. Using these conversions, we can switch between the grids as the need dictates (see Fig. 1). For example, hierarchical shapes resulting from a refinement of quads are very simple as opposed to hexagons that are not congruent (this means that it is not possible to completely cover a hexagon by a set of complete and disjoint smaller hexagons). As a result, we can convert hexagonal grids to quadrilaterals to design an efficient data structure for hexagonal grids and benefit from the simple hierarchical shape of quads and convert them back when cells with better sampling rate or a uniform neighborhood definition are desired.

Hierarchy among the cells is typically provided by refinements. Refinements introduce more cells and vertices into a model. When a refinement is applied to a cell with area  $A$ , it divides the cell into some smaller cells with area  $\frac{A}{i}$ . Such a refinement is called 1-to- $i$  refinement or a refinement with the factor of  $i$  [9]. Refinements are useful when  $i$  is an integer number since after two levels of subdivision the cells are simply scaled by an integer number (although lattices may not be aligned). However, these refinements are typically specified for a particular grid. For instance, quadrilateral 1-to-3 refinement has not been defined while triangular 1-to-3 refinement has been successfully employed in  $\sqrt{3}$  subdivision. Using hierarchical grid conversions, we propose a framework to define such refinements and study their properties.

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**Fig. 1.** Semiregular hexagonal and triangular models ((a) and (b) at the bottom) are created by hexagonal and triangular refinements ((a) and (b) at the top). Using conversions such as pairing ((c) at the top), or dual ((d) at the top), we can use simple quadrilateral hierarchical shapes and efficient data structures for packing models in (a) and (b) at the bottom into quadrilateral patches ((c) and (d) at the bottom).

## Contributions

Our main contribution is to present hierarchical conversions between regular grids. To demonstrate the usefulness of these conversions, we use them to define new near-regular and regular refinements for grids and extend an existing patch-based hierarchical data structure – Atlas of Connectivity Maps (ACM) – [10,11] to support hexagonal grids and more variety of regular and near-regular refinements.

## 2. Related work

As we present hierarchical grid conversions in this paper and use them to define new refinements and hierarchical data structures, we can categorize the work related to our method into three groups: conversions between regular grids, refinement and subdivision, and data structures proposed to support multiresolution (hierarchy) of semiregular models. In the following, we provide prior work of each group.

### 2.1. Conversion between regular grids

Grid conversion is already well explored within the Computer Graphics community, under the topic of remeshing. Triangulation [12,13] and quadrangulation [14] convert arbitrary meshes to those with cells, of triangles and quadrilaterals respectively. This remeshing may improve rendering time, mesh quality, or fulfill geometric or aesthetic constraints. Hexagonal remeshing occurs for architectural reasons or to better represent features on the mesh due to the better sampling property [15–18]. Alternatively, conversions can occur through duality remeshing to achieve a specific cell type [8], or improve smoothness [19].

These cell conversions mostly take complicated geometric properties (e.g. Gaussian curvature) into consideration for converting one type of grid to another as their applications need to satisfy a specific geometric property [14]. However, we convert the grids on 2D domains by simple operations that only change the connectivity of vertices. Some of these conversions are very straightforward. However, we combine them with refinements to define new refinements and design efficient hierarchical data structures.

### 2.2. Refinements

Regular refinements in surface modeling are the process of splitting faces into a set of smaller faces. After refinements, more faces and vertices are created and a higher resolution model is obtained. As a result, refinements can create a hierarchy of objects at different resolutions (i.e. the level of refinement). Regular refinements have many usages in computer graphics such as subdivision in which faces are initially split by a regular refinement and then vertices are geometrically modified to obtain a smooth surface.

Regular refinements are defined differently in literature. Guskov et al. [20] consider only the dyadic refinement as a regular quadrilateral refinement in which a face is split into four faces (Fig. 2(b)). Weiss and De Floriani [9] also consider the same definition for triangular faces. This type of refinement is the most common refinement as it is employed in designing popular subdivision methods such as Catmull–Clark and Loop [4,2] and useful data structures such as quadtrees [5].

Velho in [21] defines regular refinements for quadrilateral meshes as a process that produces a finer set of similar faces that are only scaled. He then categorizes regular refinements as *primal* and *dual*. In a primal subdivision, the vertices of the coarse tessellation are preserved and old edges are divided and reconnected while in a dual subdivision, new vertices are inserted in the interior of

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