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Constructive generation of the medial axis for solid models☆

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HIGHLIGHTS

- A novel constructive method is given based on reuse of the existing MA.
- Only the MA in the local re-dilation region needs to be regenerated.
- Some new properties of MA are analyzed for the constructive MA generation.
- The local re-dilation region is incrementally determined for Boolean subtraction operations.

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ABSTRACT

The medial axis (MA) is a simplified representation of complicated models which is widely used in current research. However, the efficient generation of the MA for complicated solid models continues to pose a challenge. In this study, a constructive approach for the generation of the MA is proposed for solid models after they are voxelized. With this method, the MA of the model constructed from two operand models via a Boolean operation is efficiently generated by merging the MAs of the operand models in a certain way, instead of regenerating them from scratch. To support the proposed method, the affected region of the resultant model is computed first using a Boolean operation. Second, only the MA in the affected region is regenerated by distance dilation. Third, the complete MA of the resultant model is constructed by combining the newly generated MA with the unchanged MAs of the operand models. In this study, the accuracy and complexity are analyzed for the final MA and some examples are given to illustrate the performance of the proposed method.

1. Introduction

The medial axis (MA) of an object is the set of all points having more than one closest point on the object's boundary [1]. It is useful in many engineering fields because of its beneficial properties [2]. Much research has been conducted regarding the generation of MAs [3–5]. However, the computational efficiency is still not satisfactory for complicated models, even though some strategies have been proposed for special types of models [6]. One common characteristic of almost all the existing approaches is that the model is regarded as one single part. In that case, the MA is generated each time from scratch, even when only a small modification is made to the model, which induces an intolerably low computational inefficiency. The efficient generation of the MA of solid models is still a challenge in engineering applications. Complicated engineering models often are constructed progressively using Boolean operations. Therefore, the MAs for these models can be obtained incrementally and efficiently by combining the MAs of the operand models.

Based on the above analysis, a constructive method is proposed which generates the MA of the resultant model obtained by a Boolean union or subtraction operation efficiently. In this method, the region that will be changed, hereafter called the affected region, is analyzed first. Then, the double queues based distance dilation (DQDD) algorithm is applied to the affected region. The output of the DQDD algorithm, the Euclidean distance transformation (EDT), formed by the distance values of the voxels of the model, is computed by distance dilation from outside to inside. Finally, the MA is generated using the computed EDT. The distance value of a voxel is calculated between the centers of the voxel and its touch voxel. Here, the touch voxel of a given voxel is the nearest voxel of the given voxel on the model boundary. The proposed method can also

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be used on a single model when the affected region is the whole model. The EDT and the MA in the affected regions only need to be regenerated when Boolean operations are performed on the operand models, dramatically improving the computational efficiency.

The remainder of this paper is organized as follows. In Section 2, the related work is summarized. The motivation for the study, an overview of the proposed DQDD method and some concepts are described in Section 3. Several fundamental properties are discussed in Section 4. The construction of the voxel map and the distance update of the affected region are described in Sections 5 and 6, respectively. Section 7 describes the process of MA generation. The accuracy and complexity analysis and some example results are given in Sections 8 and 9, respectively. Finally, the conclusions and the directions for future work are given in Section 10.

2. Related work

Since H. Blum [7] introduced the MA to provide an effective description for 2D objects, there has been significant research on MA generation. The approaches can be classified into three categories: thinning, tracing and Voronoi graph-based methods [8–15]. The thinning method is most similar to that applied in this study; therefore, the research related to that method is discussed. Research on improving the computational efficiency of the MA is also reviewed. For more information about the MA, readers can refer to other works [16,17].

In the thinning method, the model is usually approximated first to easily compute the MA. The computational precision is controlled by the approximation precision of the model. L. Lam et al. introduced the thinning method [18]. L.R. Nackman put forward a method which substituted a polygon/polyhedron for the smooth boundaries and thus used the MA of the polygon as the MA of the input model [19]. Based on the diffusion of the combined waves, G.L. Scott et al. proposed a method to determine the symmetric axis of a model, which was a superset of the MA [20]. This method is very effective for 0–1 images, but the computational error is considerably greater for color images. K. Siddiqi et al. [21] proposed a vector field-based thinning method. In the approach by J. Vleugels and M. Overmars [22], voxelization was recursively performed on a space that contained MA voxels until the necessary resolution was reached. To preserve the topology of the model, G. Borgefors et al. proposed a thinning method for 3D skeletonization that can be used to compute both the surface skeleton and the curve skeleton [23]. In their method, the Euclidean distance can be represented by the local distances, and an optimal method is given to compute the minimum distance between two points [24]. G.K. Viswanathan et al. also proposed a thinning method to obtain the MA of the character [25]. In this method, points are deleted from the outer boundary layer-by-layer until a single pixel-wide skeleton is produced.

To reduce the computational complexity, many researchers have proposed various strategies. Parallelizing approaches based on GPUs have been proposed by several researchers. Using GPUs, J. Ma et al. [26] obtained the 3D MA point approximation using the nearest neighbors and the normal field. Recently, A.C. Jalba et al. improved upon their work [27]. M. Ramanathan and B. Gurumoorthy explored how to exact the MA of a model generated by a Boolean union for a 2D model [28]. The boundaries of the operand models that are not part of the union were identified and their corresponding MAs were removed. Y.C. Chang et al. tried to divide the 3D polyhedron into a set of regions by using influence shells. Then, the medial surface of each region was computed and they were combined to generate the MA for the entire model [29]. The common characteristic of these methods is that the complicated model is divided into simpler parts for the computation and then recombined after the MA computation. E.C. Sherbrooke et al. [30] built the linearly approximate segments of the curve in the direction of the tracing. Their method can be used for conveniently computing the MA of polyhedrons. Using a piecewise circular boundary conversion, O. Aichholzer et al. handled models with free-form shapes to efficiently generate the MA [31]. Moreover, with their method, convergence is guaranteed. To speed up the Voronoi graph-based MA generation method, A. Meijster et al. separated the grid points of the image into independent rows and columns; thus, it is well suited for parallelization on a shared-memory machine [32]. T. Hirata devised an efficient algorithm for each row to find the lower envelope of the minima of the set of distance functions [33]. In the incremental method proposed by R. Ramamurthy and R.T. Farouki [34], a single boundary segment was added to an existing boundary-segment set at each step, modifying the Voronoi regions of the existing boundary segments. The Voronoi region of the new segment was then constructed. After the Voronoi diagram was computed, the MA was obtained by removing certain edges of the Voronoi diagram that did not belong to the MA and by adding certain edges that belonged to the MA but were absent from the Voronoi diagram.

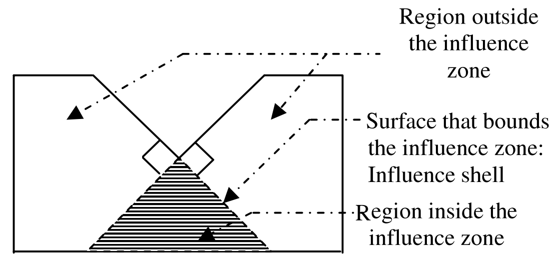

Fig. 1. The influence zone partitions an object into three regions.

3. Motivation, basic concepts and method overview

3.1. Motivation

The proposed constructive MA generation method is inspired and closely related to two ideas. The first is the division strategy proposed by Y.C. Chang et al. [29], in which the input polygon/polyhedron is divided into a set of regions, as shown in Fig. 1 (Fig. 7 in Ref. [29]). The division lies on the concave elements and the influence shells are perpendicular to the adjacent boundary elements of the concave elements. The influence zone of a concave element is bounded by the influence shells that are perpendicular to the concave element, while the area outside all influence zones constitutes other regions. After the MAs of all regions are generated, they are combined to obtain the MA of the entire model. The method can be very efficient by parallelizing the generation of the MAs of different regions. However, the existing MA cannot be reused even if only a small modification is made to the model. The reason is that the division of regions has to be redone from scratch. Moreover, the method of obtaining a robust division for a complicated 3D model is not a trivial task.

The second is the idea of piecewise MA generation followed by a combination using Boolean union operations, proposed by M. Ramanathan and B. Gurumoorthy [28]. In their work, the vanished boundary of the operand models corresponds to the area where the MA must be recomputed, as shown in Fig. 2 (Fig. 27 in Ref. [28]). It is a completely novel idea for the generation of the MA, wherein the MAs of operand models can be reused. Y.S. Liu et al. also explored a similar method [35,36]. However, the method for handling the Boolean subtraction operations is not described in their work. The difficulty in Boolean subtraction operations is that the affected region is not easily computed. The extension of the method to 3D models is also a difficult task, especially when a freeform boundary is included. M. Ramanathan and B. Gurumoorthy conducted further research in this area recently [37]. However, they only obtained the