Computer-Aided Design 62 (2015) 152–163

Contents lists available at ScienceDirect

Computer-Aided Design

journal homepage: www.elsevier.com/locate/cad

Impact of a behavior model linearization strategy on the tolerance analysis of over-constrained mechanisms☆

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HIGHLIGHTS

- A tolerance analysis approaches overview is proposed.
- A linearization procedure of the behavior model is required for both approaches.
- Some linearization strategies provide conservative probability of failure results.
- A confidence interval is obtained using two different linearization strategies.
- The order of magnitude of the probability has an effect on the convergence speed.

ARTICLE INFO

Article history:
Received 25 March 2014
Accepted 4 November 2014

Keywords:
Tolerance analysis
Behavior model
Linearization
Optimization

ABSTRACT

All manufactured products have geometrical variations which may impact their functional behavior. Tolerance analysis aims at analyzing the influence of these variations on product behavior, the goal being to evaluate the quality level of the product during its design stage. Analysis methods must verify whether specified tolerances enable the assembly and functional requirements. This paper first focuses on a literature overview of tolerance analysis methods which need to deal with a linearized model of the mechanical behavior. Secondly, the paper shows that the linearization impacts the computed quality level and thus may mislead the conclusion about the analysis. Different linearization strategies are considered, it is shown on an over-constrained mechanism in 3D that the strategy must be carefully chosen in order to not over-estimate the quality level. Finally, combining several strategies allows to define a confidence interval containing the true quality level.

1. Introduction

Geometrical tolerances influence both design functional performance and production costs, because their effects are felt at all stages of the product life cycle, so these are key elements for the design process. Appropriate design tolerances enable complex mechanical assemblies, made up of several parts, to be assembled and functional at low cost. Moreover, they enable the quality level of assemblies to be increased and ensure a high mechanical reliability of the product. To evaluate whether the design tolerances are relevant to ensure the functionality of the product, a methodology such as tolerance analysis must be applied. The tolerance analysis of mechanisms aims at verifying whether the specified design tolerances allow to reach a given quality level of the product during its design stage. The goal is to avoid the manufacturing of non-functional mechanisms. Hence, tolerance analysis is a key element [1]:

- to improve product quality,
- to reduce manufacturing costs,
- to manage and reduce waste in production.

Tolerance analysis can be divided into two approaches, whose techniques to build the behavior model are different. A comparison of both approaches is proposed in order to show their similarities and differences. Although the formulations of the mathematical models are different, both approaches need to deal with an approximated model coming from a linearization procedure in order to perform the analysis method and compute a predicted quality

☆ This paper has been recommended for acceptance by Vadim Shapiro.

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http://dx.doi.org/10.1016/j.cad.2014.11.002

level. Indeed, the analysis method is based on mathematical operations which require a linear model: a Minkowski sum and linear optimization problem with constraints. For both approaches, the linearization procedure implies simplifying the behavior model and thus modifying the accuracy of the mathematical model. This operation creates a model error which needs to be quantified. In addition, depending on the type of linearization, the corresponding error created may be different. It appears interesting to determine the best linearization procedure in order to limit the approximation error.

This paper first proposes a brief comparison of tolerance analysis approaches to show why the linearization procedure is required for both techniques. Then the paper intends to show that the linearization procedure has, of course, a real impact on the predicted quality level and on the computer time to obtain the information. However, a carefully chosen linearization procedure strategy enables this impact to be reduced. Indeed, depending on the considered strategy, the quality level may be under-estimated or over-estimated, and the computing time can be greatly increased. The analysis method must therefore take these parameters into account when applying a linearization procedure.

The next section of this paper focuses on a literature overview of both tolerance analysis approaches in order to show that a linearization procedure of the behavior model is required for all approaches. Section 3 presents the considered linearization strategies of the behavior model. The mathematical operation for the linearization of non-linear equations is detailed. Section 4 integrates the mathematical description and the solution of a tolerance analysis problem based on the model proposed by Dantan and Qureshi et al. [2,1]. Section 5 is devoted to an impact analysis of the linearization procedure on an industrial application. Results of the linearization impact are shown and discussed in this section. A conclusion ends the paper.

2. Tolerance analysis overview

Tolerance analysis aims at verifying the value of functional requirements after tolerances have been specified on each component of a mechanism. Three main issues exist [3]:

1. Modeling geometrical deviations due to the manufacturing process and modeling gaps between features.
2. Building a mathematical model to simulate the behavior of the mechanism, taking into account deviations and gaps.
3. Developing analysis methods to estimate the quality level.

2.1. Geometrical models

Modeling geometrical deviations and gaps are required in order to perform items 2 and 3. Both deviation and gap characterize a displacement between two surfaces of a mechanism. The geometry of the mechanism parts can be modeled in different ways:

- nominal surface: ideal surface whose dimensions and positions match the design.
- skin model: real manufactured surface.
- substitute surface: perfect surface associated with the skin model where the form defects are neglected.

In the present paper the form defects are omitted, so the representation of the geometrical deviations and the gaps is based on substitute surfaces. It could be between two substitute surfaces or between a substitute surface and a nominal surface [2]. Geometrical deviations (situation or/and intrinsic deviations) are modeled by random variables, written $\mathbf{X} = \{X_1, \ldots, X_n\}$. Gaps are modeled by free variables, written $\mathbf{G} = \{g_1, g_2, \ldots, g_m\}$, which need to be computed by the analysis method. Small displacements and kinematic displacements may be considered; they are used either to model small mobilities of the mechanism due to deviations and gaps, or kinematic displacements in joints.

Several representations are mentioned in the literature to deal with displacements. They can be expressed using one of the following techniques: kinematic formulation [4,5], small displacement torsor (SDT) [6,7], matrix representation [8], vectorial tolerancing [9]. The analysis method formulation is based on the small displacement torsors, see Section 4, but it is not limited to one of these techniques; all representations are suitable.

2.2. Behavior models

Building a behavior model allows to know how features of a mechanism interact, that is why relations characterizing its behavior have to be identified. In particular, these relations concern dimensional chains, in order to link features in contact with each other, with or without gaps. In addition, other relations have to be considered to prevent features from penetrating into others when there are gaps. Tolerance analysis can be divided into two distinct categories: displacement accumulation and tolerance accumulation [1]. The first category defines constraints on parameters [2,1] and the second one defines admissible volumes of variations [10–13].

- The goal of displacement accumulation is to model the influences of the deviations on the geometrical behavior of the mechanism. The relation uses the following form [14]:

$$Y = f(\mathbf{X}, \mathbf{G}) \tag{1}$$

  where Y is the response of the system (a characteristic such as a gap or a functional characteristic). The function f represents the deviation accumulation of the mechanism; it can be an explicit analytical expression, an implicit analytical expression or a numerical simulation. The difficulty in determining the function f increases with the complexity of the studied system [15,2,16].
- The aim of tolerance accumulation is to simulate the composition of tolerances i.e. linear tolerance accumulation, 3D accumulation. The admissible deviations are mapped using several vector spaces in a region of hypothetical parametric space. Tolerance accumulation uses relations between all domains to characterize the geometrical behavior. The literature mentions several techniques to represent geometrical tolerances or dimensioning tolerances, among which are T-maps® [10,17,11], gap spaces [18,19] and deviation domains [12,13].

In both cases, several types of domains and constraints are defined. Although the behavior model is based on different mathematical tools, an analogy between these types is possible. Both representations of mechanical behavior have similarities; a brief parallel of both approaches is presented in Table 1.

2.3. Tolerance analysis problem formulations

The tolerance analysis method must define a mathematical formulation able to take into account all the characteristics of the behavior model and to provide an accurate computed quality level. A comparison of the quality level formulations is presented in Table 2.

Different analysis method techniques exist, such as worst-case analysis and statistical analysis [14,2]:

- The goal of statistical tolerance analysis is to compute the probability that the requirement can be satisfied under given individual tolerances [14,24,19].
- The worst case analysis method (also called deterministic or high–low analysis method) involves defining the dimensions and tolerances such that any possible combination of workpieces provides an admissible assembly of the mechanism. In the examination of the functional requirement, the worst possible combination of each deviation is considered [25,26].