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Adaptive meshing for finite element analysis of heterogeneous materials☆

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HIGHLIGHTS

- An adaptive meshing scheme for versatile heterogeneous materials.
- A mesh adaptation algorithm based on centroidal Voronoi tessellation.
- A specific density function aiming to equalize the material variation over mesh elements.
- An adaptive sampling technique for calculating material variations.

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ABSTRACT

Adaptive meshing of *2D planar regions*, *curved surfaces* as well as *3D volumes* has been extensively studied in Finite Element Analysis (FEA) in the past few decades. Despite the maturity of these adaptive meshing approaches, most of the existing schemes assume the domain or sub-domain of interest is *homogeneous*. In the context of *FEA of heterogeneous objects*, traditional adaptive mesh generation strategies become inadequate as they fail to take into account the material heterogeneities. This paper is motivated to tackle such problems and propose an adaptive mesh generation scheme for FEA of versatile heterogeneous materials. The proposed approach takes full advantages of the material heterogeneity information, and the mesh density is formulated with a specific function of the material variations. Dual triangulation of centroidal Voronoi tessellation is then constructed and necessary mesh subdivision is applied with respect to a predefined material threshold. Experiments show that the proposed approach distributes the material composition variation over mesh elements as equally as possible and thus minimizes the number of elements in terms of the given material threshold. FEA results show that the proposed method can significantly decrease the mesh complexities as well as computational resources in FEA of heterogeneous objects and compared with existing approaches, significant mesh reduction can be achieved without sacrifice in FEA qualities.

1. Introduction

With recent technologies, design and fabrication of objects with spatially different material definitions becomes commonplace. Such objects are commonly termed as *heterogeneous material objects*. Heterogeneous material objects [1–4] possess superior properties in applications where multiple, often contradictive, functional requirements are simultaneously expected. By introducing material heterogeneities into the design domain, different properties and advantages of various materials can be fully exploited; traditional limitations due to material incompatibility/affinity problems can be naturally alleviated with gradual material variations. In the past few decades, a variety of applications have been reported in *mechanical*, *electrical*, *thermal*, *optical*, *biomedical* and other fields [5–14].

The wide applications of heterogeneous objects have aroused active research in numerical analysis of heterogeneous objects. Many Finite Element Analysis (FEA)-based approaches have been proposed for function analysis or design validities [15–21]. These methods extended traditional FEA approaches by taking the material heterogeneity into account and allowing different materials to be defined for each node or element. Most of them either use classic mesh generation schemes which result in poor accuracies (as the mesh only guarantees the geometric accuracies but fails to characterize the material heterogeneities) or alternatively,

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employ meshes with ultra-high densities to assure solution accuracies, which, on the other hand, significantly degrades the computational efficiencies [22].

Adaptive finite element mesh generation is a promising solution to such problems. In the past, numerous investigations have been conducted on adaptive mesh refinement for 2D planar regions, curved surfaces as well as 3D volumes [23,24]. Finer-resolution grids are used in regions where the surfaces exhibit large curvatures, and in planar or quasi-planar regions, sparse mesh grids are employed. However, almost all of these methods assume the components under meshing have homogeneous material compositions in the domain or sub-domain of interest. In the context of FEA of heterogeneous material objects, however, these strategies are no longer effective and directly applicable.

Motivated to take advantages of traditional adaptive meshing techniques while at the same time to incorporate the material heterogeneity into FEA studies, this paper aims to investigate effective approaches to generate adaptive meshes for heterogeneous objects. The proposed approach takes full advantages of the material heterogeneity information, and the mesh density is formulated with a specific function of the material variations. Dual triangulation of centroidal Voronoi tessellation is then constructed and necessary mesh subdivision is then applied in accordance with a predefined material threshold. Experiments show that the proposed approach distributes the material composition variation over mesh elements as uniformly as possible and thus minimize the number of elements while satisfying the material threshold requirement. Numerical results show that the proposed approach can properly balance the accuracy and computational overhead of finite element analyses and significant mesh reduction can be achieved without apparent sacrifice in FEA qualities.

2. Related work

Automatic mesh generation for FEA of homogeneous materials has been extensively studied in the past, and among others, the Delaunay triangulation methods [25,26], advancing front methods (AFMs) [27,28], Quadtree/Octree methods [29,30] are most commonly used approaches. A basic principle for automatic mesh generation schemes is the ability to construct adaptive meshes with regard to a node spacing function (or a sizing function). In general, adaptive mesh generation consists of two steps: collect information (e.g. the object geometry, a *posterior* error estimator of the solution and some economic constraints) to build a node spacing function and then construct a desirable mesh conforming to the node spacing function [30–33]. In [24], Lo provided a comprehensive review on existing adaptive meshing schemes based on node spacing functions, for instance, the *Delaunay triangulation method*, *advancing front approach*, *mesh generation using contours*, *coring technique*, *Quadtree/Octree technique* and *mesh refinement by subdivisions*.

The aforementioned methods however mainly considered the *geometric compatibility* and the *topological compatibility* of the finite element meshes. The geometric compatibility guarantees the final mesh to be closely conformable to the object shapes or geometries; and the topological compatibility ensures all the elements are properly connected with correct adjacency relationships [34].

In addition to these two compatibility requirements, Sullivan et al. [34] proposed that the *physical compatibility* should also be seriously considered, as he put it, "An accurate numerical solution requires that the domain be discretized sufficiently to describe the physics of the problem". As such, they tackled the adaptive meshing problem for heterogeneous objects, but unfortunately, they only focused on multi-material objects which are very primitive in material heterogeneity. Schimpf et al. [35] also studied the adaptive meshing problem for human organs (e.g. heart, liver, lungs), each of which is also regarded as components with "distinct" materials.

The FEA studies on Functionally Graded Materials (FGMs) have been widely investigated in recent years. Most investigations, however, did not take the local material heterogeneities into account for the meshes were usually generated with commercial software packages [36–39], which are inherently designed for homogeneous solid modeling purposes.

To our knowledge, the work done by Shin [40], perhaps, seems to be the first study on the adaptive meshing problem for FGM objects. In his work, he converted continuous material gradation into step-wise variation. Iso-material contours of the solid model were first created; triangular mesh was then generated inside each iso-material (i.e. homogeneous) region formed by iso-material contours. The advantage of this model is that it is computationally efficient, and the size of mesh elements is also adaptively determined. However, only unidirectional material gradient was taken into account in Shin's approach. No generic solutions were proposed to solve adaptive mesh generation for objects with bidirectional or even more complex material distributions [1,41,42].

Chiu et al. [43] proposed an adaptive mesh generation method for complex heterogeneous objects based on the quadtree technique. A material threshold was utilized to evaluate if a mesh element is homogeneous or quasi-homogeneous. The subdivision of the domain was recursively processed until all the elements satisfied the material threshold requirement. This method is capable of processing objects with complex material gradient functions, for instance the Heterogeneous Feature Tree (HFT) structure [42], but a large amount of computational resources are needed for geometric intersection calculations. Moreover, material compositions are evaluated at a few sampling points only (for instance, the corner points of a quadtree rectangle), and in case the material composition differences among all sampling points fall below a given tolerance, no further subdivisions will be performed any longer. Theoretically however, it is possible that abrupt material changes still exist within quadrants of interest, even though the material variations along the bounding edges are homogeneous or quasi-homogeneous. In such scenarios, Chiu et al.'s approach is incapable of generating robust and adaptive finite element meshes.

To the best of our knowledge, so far there seems to be no thorough investigations on adaptive mesh generation for heterogeneous materials. Existing studies either resort to commercial software packages, which by nature, are not suited for mesh generation of heterogeneous materials, or use unnecessarily dense meshes which introduce significant efficiency problems. This paper is motivated to bridge such a gap towards providing a generic solution to this problem. We show, in what follows, that the proposed approach can effectively generate adaptive meshes for general heterogeneous models, inclusive of simple analytic function-based as well as other complex data representations such as HFT structures.

3. Adaptive meshing of heterogeneous materials

In this section, a general scheme for adaptive meshing of heterogeneous materials is first presented. Algorithmic details on how to apply the adaptive meshing method to analytic function-based and HFT-based heterogeneous models are then elucidated with examples.

3.1. General adaptive meshing scheme

For mesh generation of heterogeneous materials, there are four important factors to be considered:

(i) Geometric approximation