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Evolutionary topology optimization of continuum structures with a global displacement control☆

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HIGHLIGHTS

- The proposed method achieves optimal designs with globally controllable deflections.
- Solutions with the proposed method can maintain aerodynamic shape when deformed.
- Economic lightweight designs are achieved with a volume minimization scheme.
- Robust and effective numerical algorithms are proposed for clear 0–1 designs.

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ABSTRACT

The conventional compliance minimization of load-carrying structures does not directly deal with displacements that are of practical importance. In this paper, a global displacement control is realized through topology optimization with a global constraint that sets a displacement limit on the whole structure or certain sub-domains. A volume minimization problem is solved by an extended evolutionary topology optimization approach. The local displacement sensitivities are derived following a power-law penalization material model. The global control of displacement is realized through multiple local displacement constraints on dynamically located critical nodes. Algorithms are proposed to secure the stability and convergence of the optimization process. Through numerical examples and by comparing with conventional stiffness designs, it is demonstrated that the proposed approach is capable of effectively finding optimal solutions which satisfy the global displacement control. Such solutions are of particular importance for structural designs whose deformed shapes must comply with functioning requirements such as aerodynamic performances.

1. Introduction

Topology optimization is a powerful tool resulting in high structural performance with great material saving. It is capable of speeding up the structural design process and producing reliable solutions for structural design problems. This area has been extensively investigated in the past three decades since the modern formulation of optimal layout theories by Prager and Rozvany [1]. Several Finite Element (FE) based methods have been developed for topology optimization of continuum structures. One most popular technique is the Solid Isotropic Material with Penalization (SIMP) method [2,3], where the isotropic material property in each element is determined by the element relative density. By penalizing the element relative density using a power-law interpolation scheme, a nearly void–solid design is expected after an iterative procedure such as the Method of Moving Asymptotes [4].

A popular group of methods for topology optimization is the Evolutionary Structural Optimization (ESO) method and its descendent versions. The ESO method was first proposed by Xie and Steven [5,6] in the early 1990s. This original version follows a straightforward algorithm of iteratively removing inefficient material and thus drives the structure to evolve towards an optimum. The bi-directional evolutionary structural optimization (BESO) method was proposed as an extended version of ESO in the late 1990s [7,8]. The BESO method allows material to be added to the most demanding places while inefficient material is removed simultaneously. The ESO/BESO methods use binary design variables and directly deliver void–solid solutions that are very desirable. The ESO/BESO methods can be easily implemented in a wide range of engineering applications such as [9–13].

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Deflection constrained optimization problems are related with stiffness optimization problems [14] to a large extent. These two sets of problems are in fact equivalent under single point loaded systems and with deflection control at the load point in the load direction. However generally, the stiffness optimal design is not the same with the optimal deflection design. The stiffness optimization increases the structural overall stiffness and thus the deflection is roughly reduced as an indirect effect. Very often in practice, it is not the stiffness but the deflection over parts of or whole of the structure that is the crucial factor. Despite vast research on stiffness optimization, the local displacement constraint on continuum structural optimization is found in relatively less literature such as [15–17].

For some structures such as an aircraft wing, the exterior surface should undergo minimal shape change under deformation in order to maintain the aerodynamic performance [18]; this requires that the displacements of the surface shall remain within a certain limit. In such cases, the displacements of a group of local nodes are of concern; or more generally, the displacement limit is addressed as a global constraint. The deflection constrained optimization problem addressed in the present paper is a global control problem. The common way of obtaining the displacement sensitivity is to apply a unit virtual load on the original model and get the displacement vector from the virtual system [16]. For global displacement control, the maximum deflections occur unpredictably at different nodes through the optimization history. Therefore the virtual loads must be built dynamically after the maximum deflections are identified in the real system at each stage; and the real and virtual systems must be analysed in two subsequent FEAs. This increases the computational cost significantly. More importantly, numerical instabilities caused by dynamical relocation of control nodes are hard to handle; as a consequence, stable solution convergence is highly difficult to achieve.

Multiple displacement constraints optimization can be found in literature such as [19–23]. The common strategy of this art is to control the local displacement at pre-defined locations, usually at the load point and in the load direction, such as in [19–21]; in other words, a modified compliance minimization problem is actually dealt with. Pre-defined displacement constraints are also applied to specific structural systems such as truss structures [22,23]. The complexity of solving multiple constraints is reduced due to the fact that less design variables are present in the design domain. Since the displacement control locations and directions are limited (by pre-definition or by the applied load), a genuine "global control" for displacements is not realized in these methods. In some cases, the critical displacement may occur at a location completely different from the load points; such fixed multi-constraint approaches will not be able to tackle the maximum displacement. Multi-constraint optimization for continuum structures is usually a highly non-linear problem that is theoretically very difficult to solve, especially in cases as the global displacement constraint that controls displacements on dynamic locations.

This paper presents a global control method for displacements of continuum structures. A topology optimization problem of volume minimization is formulated and solved by a new BESO approach. The global displacement constraint directly imposes a maximum allowable limit for nodal displacements within the whole domain or some user-specified sub-domains of the structure. The locations and directions of maximum nodal displacements are dynamically detected, the numerical instabilities of which are adaptively dealt with by robust stabilization algorithms. The subsequent sections are organized as follows: Section 2 formulates the topology optimization problem for global displacement control; Section 3 presents the sensitivity calculation for displacements; Section 4 is dedicated to the global displacement constraint algorithms for stabilizing the evolution history and solution convergence; the numerical implementation of the proposed method is outlined in this section; Section 5 shows numerical examples with discussions; the conclusions are drawn in Section 6.

2. Problem statement

The global displacement control can be realized by confining the maximum displacement of the structure. With the volume being the objective function, the problem is formulated as follows.

Minimize V,

subject to

$$V = \sum_{i=1}^{N} V_i x_i, \quad x_i \in \{x_{\min}, 1\} \tag{1}$$

$$d_j \le d_{\max} \le d^*, \quad j = 1, \ldots, L \tag{2}$$

$$\mathbf{P} = \mathbf{KU} \tag{3}$$

where V is the volume of the structure, V_i is the volume of the ith element, N is the total number of elements; x_i is the binary design variable, i.e. either 1 or 0 denoting the element status of "solid" or "void"; the design variable can also be treated as the element relative density [16] with a very small value such as 10^{-6} representing the void status; d^* is the maximum allowable displacement value, $d_{\max}$ is the maximum displacement in the structure under loading, and d_j is the displacement of the jth element within a prescribed domain Ω_{disp} where the displacement constraint is active. Eq. (3) is the static equilibrium with $\mathbf{P}$ as the applied load vector, $\mathbf{K}$ as the global stiffness matrix and $\mathbf{U}$ as the global displacement vector.

The displacements above can be addressed in any certain directions, or can be treated as relative displacements simply when some nodes in Ω_{disp} are fixed. Constraining the relative displacements is of particular importance when the deformed shape is concerned.

3. Displacement sensitivity calculation

3.1. Material interpolation scheme

Material interpolation schemes usually express the material properties as a function of the design variables in order to facilitate the sensitivity analysis. In the power-law material model SIMP [24], the Young's modulus of an element is determined by the element relative density through the following penalization formulation.

$$E(x_i) = x_i^p E^0 \tag{4}$$

where E^0 is the Young's modulus of the solid material and p denotes the penalty exponent which is usually set as $p \ge 3$. Note that the Poisson's ratio is usually assumed to be irrelevant in SIMP. Therefore, the stiffness matrix can be expressed in a similar way as follows.

$$\mathbf{K} = \sum_i x_i^p \mathbf{K}_i^0 \tag{5}$$

where $\mathbf{K}_i^0$ denotes the stiffness matrix for the solid element, i.e. when $x_i = 1$.

3.2. Displacement sensitivity in axial directions

The displacement kth component can be obtained by multiplying the displacement vector with a unit virtual load vector $\mathbf{F}^k$, of which the kth component is unity while all other components are zero.

$$u^k = \mathbf{F}^k \cdot \mathbf{u} \tag{6}$$

$$\mathbf{F}^k = \{0, 0, \ldots, 1, 0, \ldots\} = \{f_1, f_2, \ldots, f_k, f_{k+1}, \ldots\}. \tag{7}$$

With the virtual load $\mathbf{F}^k$ and the applied load $\mathbf{P}$ being constant, differentiating the kth displacement component with respect to