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Leading a continuation method by geometry for solving geometric constraints

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ABSTRACT

Geometric constraint problems arise in domains such as CAD, Robotics, Molecular Chemistry, whenever one expects 2D or 3D configurations of some geometric primitives fulfilling some geometric constraints. Most well-constrained 3D problems are resistant to geometric knowledge based systems. They are often solved by purely numerical methods that are efficient but provide only one solution. Finding all the solutions can be achieved by using, among others, generic homotopy methods, that become costly when the number of constraints grows. This paper focuses on using geometric knowledges to specialize a so-called coefficient parameter continuation to 3D geometric constraint systems. Even if the proposed method does not ensure obtaining all the solutions, it provides several real ones. Geometric knowledges are used to justify it and lead the search of new solutions.

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1. Introduction

Solving geometric constraints is addressed in many fields such as CAD, molecular modeling, robotics. It consists of finding positions of geometric entities such as points, lines, circles in 2D and also planes and spheres in 3D. These entities must satisfy some constraints related to distances, angles, tangencies, incidences, distance ratio and so on. Moreover, most softwares of geometric modeling and simulation incorporate constraint solvers to allow the user to define objects by constraints. Constraint systems are usually considered to be well-constrained in the sense that there exists a finite number of solutions up to rigid body motions. Indeed rotations and translations have no effect on the satisfaction of the considered constraints.

This issue has been widely studied in the case of 2D design in CAD. Several approaches have been described in literature, see [1,2] for recent surveys. Basically, algebraic approaches translate the problem into equations and then use algebraic techniques. Geometric approaches yield solutions by means of classical theorems or constructions of geometry. Solvers often perform first a decomposition of the problem into sub-problems easier to solve. In addition, solving the whole problem is faster. The decomposition is based on the geometric nature of the statements. We can notice that some geometric methods as well as decomposition processes rely on basic geometric constructions, allowing efficient implementations through graph algorithms.

Solving 3D geometric constraint systems is much more difficult particularly if all the solutions of the systems, assumed well-constrained, are wanted. Indeed, geometric methods or decomposition processes used in 2D cannot be easily extended in 3D. This is illustrated for instance in rigidity theory where a combinatorial characterization of rigidity is known for 2D realizations of graphs but not for 3D ones. Considering decomposition, most problems related to 3D polyhedrons are not decomposed by usual combinatorial methods based on graphs. This is why industrial software products are usually based on numerical methods such as Newton–Raphson or Homotopy. Unlike the Newton–Raphson method, methods that use homotopy could propose several solutions. This is a reason why they have been particularly studied for solving geometric constraints:

(i) in [3], a classical homotopy approach is used which is able to find all the solutions. But yielding all the solutions is very inefficient.
(ii) in [4], a specialized version of homotopy is used where the sketch given by the designer is a starting system. In this case, only one solution could be provided because the goal is to find the solution closest to the sketch.

In this paper, we design a homotopy based method specialized in 3D geometric constraints solving. In this field several features must be considered. First, the number of solutions can be exponential with the number of constraints. It is therefore not necessary to produce all solutions at once but only some of them. Indeed, some methods yield one solution that may not be the one expected by the user. Next, the user provides a sketch that should be used to guide the search for similar solutions. Finally, the solutions must be provided within a reasonable time for the user that is

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Unknowns:
point $P_0, ..., P_5$
Parameters:
length $h_0, ..., h_8$
Constraints:
$distance(P_0, P_1) = h_0$
...
$distance(P_2, P_4) = h_8$
$coplanar(P_0, P_1, P_2, P_3)$
$coplanar(P_0, P_3, P_4, P_5)$
$coplanar(P_1, P_2, P_4, P_5)$

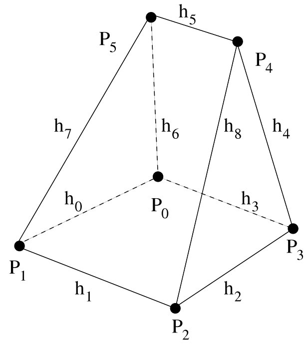

Fig. 1. A symbolic statement (left part) and a dimensioned sketch (right part) of the pentahedron problem. Edges are distances constraints of parameters h_i.

to say less than 15 s. Our approach takes into account all these features and quickly generates solutions to many problems of 3D constraints. The sketch and a plan for geometric construction, obtained by reparametrization, are used to control the search for solutions.

The rest of the text is structured as follows. Section 2 introduces notations about geometrical constraint systems. Basics on homotopy methods are given in Section 3 before outlining our method. Section 4 discusses specific homotopy paths and how they help in converging towards solutions. Section 5 addresses the issue of degenerated cases for instance involving coplanar faces. Section 6 introduces construction plans and their contribution in paths tracking. Sections 7 and 8 give some results and prospects.

2. Geometric constraint solving problems and construction plans

2.1. GCSP

In this paper, a Geometric Constraint Solving Problem (GCSP) is denoted by $G = (C, X, A)$ or also $C[X, A]$. It consists of a set C of constraints on a set X of geometric objects, depending on parameters A. A GCSP is defined over a geometric universe which specifies the dimension of the space where constructions are made, the types of geometric objects (points, circles, planes, *etc.*) and types of constraints (distances between two points, coplanarity of four points, *etc.*).

The 3D GCSP shown in Fig. 1 represents a well-constrained pentahedron with points $P_0, \ldots, P_5$. Among constraints some are *dimensioned* constraints such as distances, others are *Boolean* constraints such as coplanarity constraints. A numerical value for $x \in X \cup A$ is a *valuation* of x and is denoted $\sigma(x)$. For instance, in 3D, $\sigma(P_0) = (0, 0, 0)$ is a valuation of P_0. The notion of valuation is extended to any subset Y of $X \cup A$; a valuation $\sigma(Y)$ is a set of valuations for elements of Y.

2.2. Solutions

For a valuation of parameters $\sigma(A)$, a valuation $\sigma(X)$ is a *solution* (or a *figure*) of $G = C[X, A]$ if all the constraints c_i of G are satisfied.

A 3D dimensioned sketch of the pentahedron problem depicted in Fig. 1. This sketch is itself a valuation $\sigma_{\text{sk}}(X)$, and if it satisfies coplanarity constraints, it is a solution of G for some values $\sigma_{\text{sk}}(A)$ that can be read on the sketch.

Notice that if $\sigma(X)$ is a solution of $G = C[X, A]$, all solutions obtained by rigid body motions (translations and rotations) of $\sigma(X)$ are solutions of G since constraints are invariant under rigid body motions. The set of all solutions of G can be partitioned into equivalence classes such that two figures are in the same class if and only if they correspond each others by a rigid body motion. Only one solution of each class is relevant and some coordinates (6 in 3D) are fixed: we say that solutions are sought up to rigid body motions. In the pentahedron problem, one can fix the three coordinates of P_0, two coordinates of P_1 and one of P_2.

2.3. Construction plans

A construction plan $\text{CP}[O, A_p]$ is a sequence of terms $o_i = f(o_1, \ldots, o_{i-1}, A_p)$ with O a set of geometrical objects and A_p a set of parameters. It expresses a geometric construction where object o_i is built from previously built objects. Functions f usually compute loci intersections such as circle–line, circle–circle, sphere–sphere, and so on. Construction plans are commonly used in *geometric solver* and were introduced in [5] for instance to express symbolic solutions.

For a given GCSP, a geometric solver tries to produce a construction plan. Such a plan can then be evaluated numerically when numerical values are given for the parameters. Note that, some functions provide several values for results. For example, the intersection of two circles can contain two points. Therefore, numerical evaluation of a construction plan gives rise to a tree of solutions. Each branch corresponds to a solution. The benefit of having a construction plan is the ability to produce several or all solutions.

Unfortunately, most of the 3D examples from CAD cannot be fully treated by such geometric solvers. A method presented in [6] proposes to transform a GCSP by removing some numerical constraints and replacing them with others in order to make possible the creation of a construction plan. It is therefore a method of reparametrization of GCSP. In a second phase, numerical values for the added distances and angles are sought in order to satisfy the removed ones. This approach is effective when the number of added constraints is one or two, but is very expensive otherwise (see however [7]).

Our method is used for cases in which the geometric solvers fail or are not efficient enough. However, we will use a construction plan to guide our homotopy method. This plan will be obtained by reparametrization but it will not be to find values for the added parameters. It will be used to test some properties of solutions and to discover paths leading to new solutions.

2.4. Reading solutions

Homotopy methods proceed by deforming initial solutions. In our case, an initial solution is formed by the sketch on which the values for variables in $A \cup X$ are read. Values of unknowns are easily obtained from the sketch but the values of parameters must be calculated.

For each constraint $c(x_1, \ldots, x_n, a)$ where x_i is an unknown and a is a parameter, a function $a = f(x_1, \ldots, x_n)$ is assumed that computes the value of parameter a. For instance, for a constraint of distance between two points, function f simply calculates the Euclidean distance between two points.

In our method, getting parameter values is useful in various ways. Indeed, at key moments a construction plan is consulted. This construction plan comes from a reparametrization and corresponds to parameter values that slightly differ from the initial system. These values have then to be read from the current deformed figure.

3. Homotopy solving

Our method adapts a usual continuation method whose principles are summarized in the following.

3.1. Continuation methods

The goal of a classical homotopy resolution is to find the roots of a function $F : \mathbb{K}^m \to \mathbb{K}^m$. Roughly speaking the method starts

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