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Optimal analysis-aware parameterization of computational domain in 3D isogeometric analysis

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ABSTRACT

In the isogeometric analysis framework, a computational domain is exactly described using the same representation as the one employed in the CAD process. For a CAD object, various computational domains can be constructed with the same shape but with different parameterizations; however one basic requirement is that the resulting parameterization should have no self-intersections. Moreover we will show, with an example of a 3D thermal conduction problem, that different parameterizations of a computational domain have different impacts on the simulation results and efficiency in isogeometric analysis. In this paper, a linear and easy-to-check sufficient condition for the injectivity of a trivariate B-spline parameterization is proposed. For problems with exact solutions, we will describe a shape optimization method to obtain an optimal parameterization of a computational domain. The proposed injective condition is used to check the injectivity of the initial trivariate B-spline parameterization constructed by discrete Coons volume method, which is a generalization of the discrete Coons patch method. Several examples and comparisons are presented to show the effectiveness of the proposed method. During the refinement step, the optimal parameterization can achieve the same accuracy as the initial parameterization but with less degrees of freedom.

1. Introduction

Usually, CAD modeling software relies on splines or NURBS representations, while the (physical) analysis software for CAD objects uses mesh-based geometric descriptions (structured or unstructured). Therefore, in conventional approaches, several information transfers occur during the design phase, yielding approximations and non-linear transformations that can significantly deteriorate the overall efficiency of the design optimization procedure.

The isogeometric analysis (IGA for short) approach proposed by Hughes et al. [1] aims to overcome this difficulty by using CAD standards as unique representation for all disciplines. For 3D analysis problems, the isogeometric approach consists in developing methods that use NURBS representations for all design and analysis tasks:

- the geometry is defined by NURBS surfaces;
- the computation domain is defined by trivariate NURBS volumes instead of discrete meshes;
- the solution fields are obtained by using a finite-element approach that uses NURBS basis functions instead of classical Lagrange polynomials;
- the optimizer directly controls NURBS control points.

This framework allows us to compute the analysis solution on the exact geometry (not a discretized geometry), obtain a more accurate solution (high-order approximation), reduce spurious numerical sources of noise that deteriorate convergence, and avoid data transfers between the design and analysis phases. Moreover, NURBS representation is naturally hierarchical and allows us to perform refinement operations to improve the analysis result.

In finite element analysis (FEA for short), mesh generation generates a discretized geometry as a computational domain from a given CAD object; it is a key and the most time-consuming step. In the IGA framework, the parameterization of the computational domain, which corresponds to the mesh generation in FEA, also has some impact on the analysis result and efficiency. Moreover, while in FEA, arbitrary refinements on the computational mesh can be performed, in IGA using tensor product B-splines, the refinement is not arbitrary: only refinement operations in the ξ direction, η

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direction and ζ direction by knot insertion or degree evaluation can be performed. Hence, in IGA, the parameterization process of the computational domain is an important task.

The parameterization of a computational domain in IGA is determined by control points, knot vectors and the degrees of B-spline objects. For three dimensional IGA problems, the knot vectors and the degrees of computational domains are determined by the given boundary surfaces. Hence, finding the optimal placement of inner control points for a specified physical problem is a key issue in IGA. A basic requirement of resulting parameterization for IGA is that it should not have self-intersections. In this paper, we first propose a linear and easy-to-test sufficient condition for the injectivity of trivariate B-spline parameterization. Then we show that different parameterizations of a computational domain have different impacts on the simulation results in IGA. For problems with exact solutions, a shape optimization method is proposed to obtain an optimal parameterization of the computational domain. A discrete Coons volume method is proposed to construct an initial trivariate B-spline parameterization from the given boundary surfaces. Some examples and comparisons are presented based on the heat conduction problem to show the effectiveness of the proposed method.

The remainder of the paper is organized as follows. Section 2 reviews the related work in isogeometric analysis. Section 3 proposes linear sufficient conditions for the injectivity of a trivariate B-spline parameterization. Section 4 describes a test IGA model and shows the impact of different parameterizations of a computational domain. Section 5 presents a discrete Coons volume method to construct an initial trivariate B-spline parameterization from boundary surfaces, and our shape optimization method to obtain an optimal parameterization of a computational domain. Some examples and comparisons are also presented in Section 5. Finally, we conclude this paper and outline future works in Section 6.

2. Related work

In this section, we review related works in IGA and the parameterization of computational domains.

The concept of IGA was firstly proposed by Hughes et al. [1] in 2005 to achieve the seamless integration of CAD and FEA. Since then, many researchers in the fields of mechanical engineering and geometric modeling have been involved in this topic. The current work on isogeometric analysis can be classified into three categories: (1) application of IGA to various simulation problems [2–10]; (2) application of various geometric modeling tools to IGA [11–14]; (3) accuracy and efficiency improvement of IGA framework by reparameterization and refinement operations [15–21].

The topic of this paper belongs to the third field. As far as we know, there are few works on the parameterizations of computational domains for IGA. Martin et al. [21] proposed a method to fit a genus-0 triangular mesh by B-spline volume parameterization, based on discrete volumetric harmonic functions; this can be used to build computational domains for 3D IGA problems. A variational approach for constructing NURBS parameterization of swept volumes is proposed by Aigner et al. [15]. Many free-form shapes in CAD systems, such as blades of turbines and propellers, are covered by this kind of volume. Cohen et al. [17] proposed the concept of *analysis-aware modeling*, in which the parameters of CAD models should be selected to facilitate isogeometric analysis. The approximate implicitization technique is used for the parameterization of computational domains in [22]. In [23], we consider optimal parameterizations of planar domains for 2D isogeometric analysis. In this paper, we extend this approach to volumetric problems. A method for generating optimal analysis-aware parameterizations of computational domains for 3D IGA problems is proposed based on a shape optimization method.

3. A linear sufficient condition for injectivity of planar B-spline parameterization

The main idea of the isogeometric approach is to use the same representation for the geometry and the physical solutions we are interested in. Schematically, the geometry Ω involved in the physical problem can be a parametric volume in a three-dimensional space $\mathbb{R}^3$. Let us call $\mathbf{x} = (x, y, z)$ the coordinates associated to this space. In our case, this geometry will be represented by a parameterization σ for a domain $\mathcal{P}$ of the parameter space. Let us call $\mathbf{u}$ the coordinates of this parameter domain, which could be of dimension 3 for a volume. This parameterization will be given by B-spline functions with knots in $\mathcal{P}$ and control points in $\mathbb{R}^3$.

The concept of isogeometry consists in representing the physical quantities $\Phi \in \mathbb{R}^p$ on the geometry Ω using the same type of B-spline representation as for the geometry Ω. In other words, given a point $\mathbf{x} = \sigma(\mathbf{u}) \in \Omega$ with $\mathbf{u} \in \mathcal{P}$, we associate to it the physical quantities $\Phi(\mathbf{u})$ where $\Phi(\mathbf{u})$ is a B-spline function with nodes in $\mathcal{P}$ and control points in $\mathbb{R}^p$. This means that the map $\mathbf{x} \in \Omega \mapsto \Phi \in \mathbb{R}^p$ is defined *implicitly* as $\mathbf{x} \mapsto \Phi \circ \sigma^{-1}(\mathbf{x})$.

Consequently, the framework of isogeometry is thus valid when the parameterization σ of the geometry is *injective* (or bijective on its image). We are going to describe sufficient and easy-to-check conditions for the injectivity of σ. We will consider this problem in the context of finding a "good" parameterization of a domain when its boundary is given. In [24], a general sufficient condition is proposed for injective parameterization.

Proposition 3.1. *Suppose that σ is a C^1 parameterization from a compact domain $\mathcal{P} \subset \mathbb{R}^n$ with a connected boundary to a geometry $\Omega \subset \mathbb{R}^n$. If σ is injective on the boundary $\partial\mathcal{P}$ of $\mathcal{P}$ and its Jacobian J_σ does not vanish on $\mathcal{P}$, then σ is injective.*

For a parameterization σ from $[a, b] \times [c, d] \times [e, f]$ to $\Omega \subset \mathbb{R}^3$, we define the boundary surfaces as the image of $\{a\} \times [c, d] \times [e, f]$, $\{b\} \times [c, d] \times [e, f]$, $[a, b] \times [c, d] \times \{e\}$, $[a, b] \times [c, d] \times \{f\}$, $[a, b] \times \{c\} \times [e, f]$, $[a, b] \times \{d\} \times [e, f]$, by σ. We say that σ defines a *regular boundary* if these surfaces do not intersect pairwise, except at their end curves and if they have no self-intersection.

As a consequence of the previous proposition, we get the following injectivity test for standard trivariate B-spline parameterizations of a 3D computational domain.

Proposition 3.2. *Let σ be a C^1 parameterization from $[a, b] \times [c, d] \times [e, f]$ to $\Omega \subset \mathbb{R}^3$ which defines a regular boundary. If its Jacobian J_σ does not vanish on $[a, b] \times [c, d] \times [e, f]$, then σ is injective.*

These tests involve injectivity conditions on the boundary, which can be checked recursively using the same techniques, non-intersection tests for boundary curves and surfaces which are provided for instance by geometric (subdivision) algorithms and the local injectivity condition corresponding to the non-vanishing of the Jacobian. This last condition requires us to test on all the domain Ω that the Jacobian does not vanish. Hereafter we propose a sufficient and easy-to-test condition to ensure the local injectivity condition.

We consider the case of a trivariate parameterization

$$\sigma : \mathbf{u} \in \mathcal{P} := [a, b] \times [c, d] \times [e, f] \mapsto \sigma(\mathbf{u}) := \sum_{0\le i\le l_1, 0\le j\le l_2, 0\le k\le l_3} \mathbf{c}_{i,j,k} N_{i,j,k}(\mathbf{u}),$$

where $\mathbf{c}_{i,j,k} \in \mathbb{R}^3$ are the control points and $N_{i,j,k}(\mathbf{u})$ are the B-spline basis functions. The derivative of $\sigma(\mathbf{u})$ with respect to $\mathbf{u}_1$ can be