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Complete swept volume generation, Part I: Swept volume of a piecewise C^1-continuous cutter at five-axis milling via Gauss map

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ABSTRACT

In this paper, we present a methodology to generate swept volume of prevailing cutting tools undergoing multi-axis motion and it is proved to be robust and amenable for practical purposes with the help of a series of tests. The exact and complete SV, which is closed from the tool bottom to the top of the shaft, is generated by stitching up envelope profiles calculated by Gauss map.

The novel approach finds the swept volume boundary for five-axis milling by extending the basic idea behind Gauss map. It takes piecewise C^1-continuous tool shape into account. At first, the tool shape is transformed from Euclidean space into Tool map (T-Map) on the unit sphere and the velocity vector of a cutter is transformed into Contact map (C-Map) using Gauss map. Then, closed intersection curve is found between T-Map and C-Map on the Gaussian sphere. At last, the inverse Gauss map is exploited to get envelope profile in Euclidean space from the closed curve in the range. To demonstrate its validity, a cutting simulation kernel for five-axis machining has been implemented and applied to mold and die machining.

1. Introduction

In modern production facilities, there is an increased demand for product quality and cost reduction. At the same time, there is an increase in complexity of part geometries and machining operations. In order to be able to manufacture with a high degree of quality, economy and flexibility, simulation systems are widely used and are securing their place in modern manufacturing processes. One of them is the NC verification system which simulates numerical control (NC) milling process, which is one of the most prevailing manufacturing processes. During milling process, the blank material is removed by geometrically defined cutters which move along the given tool paths. At the end of the process a target part is achieved. To ensure product quality and to avoid junks resulting from overcut or collision between the tool and the material, it is necessary to simulate the milling process before real cutting. Because the time-to-market is shortening in modern production cycles, the verification of milling process is becoming a more crucial stage in the whole manufacturing [1].

The ultimate goal of milling simulation is to predict results of the workpiece as realistically as possible prior to machining, not only with macrosurface information but also with microsurface information. For instance, the in-process workpiece is categorized into the macrosurface information; however the accuracy of macrosurface mainly depends on the exactness of the swept volume (SV) of a moving cutter and the representation model of the workpiece in space. Furthermore, it is difficult to calculate if the cutter undergoes both rotation and translation motion. Since a series of partial differential equations, including trigonometric functions, is to be solved, it is not usually possible to calculate the exact SV within an acceptable execution time. Besides this, SV should be continually subtracted from the workpiece geometry in order to enable verifying NC programs. Usually the volume subtraction is realized with ease through the CSG (Constructive Solid Geometry) operation for three-axis machining but it is quite a challenging problem for simultaneous five-axis motion [2]. Accordingly, the SV of a cutter undergoing five-axis motion is represented approximately by the sum of pure translation volume and pure rotational volume in the NC simulation tools [3–5].

2. Related work

Several fundamental developments in the past decades have led to a better understanding of general SVs. The underlying formulation for characterizing the generated SV has appeared under different names in various fields and context. Some examples are: the determination of collision-free trajectory between the moving objects and static obstacles [6], the design of moving arms for robots [7], the representation of the boundary of moving objects for visualization and computer graphics [8], the formulations for modeling sweep procedure [9], and the calculation of the volume's mass properties for solid modeling [10].

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There are also several methods for formulating SV for a cutting tool. The geometry of the SV can be extremely complex if the orientation of the tool axis changes continually. In fact, such geometry is out of the range of the modeling capacity of the conventional solid modelers [11,12] which are based on the CSG representation technique, because the SV cannot be represented with prismatic geometries such as cylinders, cones and spheres, etc., when the cutter undergoes simultaneous five-axis motion or self-penetration (SP) of the tool body occurs. Their classification depends on accuracy, robustness and simplicity, and they are classified into discrete or analytic, approximate or exact SV representation. In the following subsections, relevant approaches are categorized and explained in light of their strengths and weaknesses. Some of them are being successfully applied in practice.

2.1. Envelope theory and tangency condition

Wang et al. [13] present a method for modeling SVs of the tool described in moving frame by computing the tangency condition that the Jacobian determinant vanishes [7], or geometrically speaking, the instantaneous velocity $\boldsymbol{v}(P)$ at a surface point P of the generator is perpendicular to the surface normal $\boldsymbol{n}(P)$ at P [14–16]:

$$f = \boldsymbol{v}(P) \cdot \boldsymbol{n}(P) = 0, \quad P \in \Phi(t), t \in \mathbb{R}. \tag{1}$$

Based on this tangency condition (Eq. (1)), trigonometric differential equations are solved to find the time-dependent envelope curves. The assumptions of this approach is, however, that (1) the trajectory of the cutting tool is piecewise differentiable, and (2) the boundary of the generator is a regular surface, which means that it has no sharp points or edges. Moreover, Wang et al. have not handled the degenerate case: that is, self-intersection of the cutter.

Chiou et al. [17,18] introduce the 3D shape-generating profile method, which uses the tangency condition f of the surface normal $\boldsymbol{n}$ at the tool surface point P and the moving direction vector $\boldsymbol{v}$ of the cutter which is modeled with 7 parameters [19], at a cutter contact (CC) point. The deficiency of this approach, however, is that the constant cutter velocity $\boldsymbol{v}$ has been used. It might be computationally simple, but this method is not correct because the cutter velocity is not constant, but varies continually according to the location on the cutter surface. Therefore, it intrinsically includes the computational fault. The three-dimensional shape-generating profile method was revised and later published in Ref. [20]. In this revision, the machine configuration and tool movements defined in the NC program are considered, and the tangency condition $f = \boldsymbol{n}(P) \cdot \boldsymbol{v}(P) = 0$ is upgraded, which consists of the surface normal $\boldsymbol{n}(P)$ and the movement vector $\boldsymbol{v}(P)$ of an arbitrary point P on the cutter surface.

A similar work to Wang et al. [13] and Chiou et al. [17] has been done by Du et al. [21–23]. In this work, they adopt the method of moving frame introduced by [13], as well as the method of explicit closed-form representation, introduced by [17], to formulate envelope profile for five-axis tool motions. The partial extension is dedicated to solution analysis and special case analysis of generalized cutters for five-axis tool motions by using rigid body motion theory [24] and envelope theory [25]. SV is modeled in the framework of an open source [26]. Because SV and raw stock are represented by non-uniform rational B-spline (NURBS) surfaces, whereas cutting simulation is conducted by Boolean subtraction between SV and raw stock, the execution time is too expensive to realize the machining simulation under a proper cycle time.

2.2. Sweep-envelope differential equation (SEDE)

Blackmore et al. [27,28] introduce the sweep-envelope differential equation (SEDE) in order to overcome the deficiency of envelope theory [29]. The fundamental idea behind it is that SV of the cutter may be formed by some self-intersecting envelope surfaces and that envelope theory is essentially local in nature. SEDE requires the existence of a object $M(t) := \{\mathbb{x} \mid f(\mathbb{x}, t) = 0\}$ moving along the trajectory $\sigma_t(\mathbb{x}) = \xi(t) + A(t)\mathbb{x}$ in the fixed frame, where $\mathbb{x} = (x, y, z)$, t is the time; $\xi(t)$ a position vector and $A(t)$ an orthogonal 3×3 matrix. The object M is represented with the implicit equation f, which is to be closed and bounded with a smooth boundary surface ∂M. That is to say, f is negative in the interior of M, zero on ∂M and positive in the exterior of M. Usually, a smooth function f characterizing an object M is not *a priori*. Hence the smooth approximation of piecewise smooth boundary surface ∂M_i for the ith tool component is provided by implicit polynomial equations for which parameters should be determined empirically and elaborately.

The power of the SEDE method is that the main portion of the SV boundary can be generated consequently once the orbits of the initial grazing points of an object are obtained by solving the SEDE. The disadvantages of this approach, however, are that the SEDE is based on complex mathematical backgrounds, such as partial differential equations, advanced vector calculus, etc. Moreover, since the initial grazing points are calculated after the cutter is triangulated (or approximated) under a certain given tolerance, initial grazing points are displaced from the actual cutter surface, which in turn affects the accuracy of the SV calculated.

2.3. Imprint method

Sheltami et al. [30] regard the SV of toroidal cutters rotating about a point on the extension of the tool axis as the superposition of "generating circles", which are circular ends at the bottom of toroidal cutters. The side of the cutter, which is made up of sectional curves of the tool surface above these circles is added to the superposition of generating circles in order to generate the swept surface of the moving cutter. These side and bottom curves represent imprints of the cutter onto a workpiece surface. It is assumed that during the tool movement the tool axis remains strictly on the feed plane that includes the feed direction and the tool axis, and that the type of cut is a pure rotation in the feed plane along the feed direction. Therefore the general motion type which involves twists and turns in space is not dealt with and the practical application of the "generating circles" technique is thus severely limited.

Roth et al. [31,32] build further upon concepts developed by Sheltami et al. [30]. First, they observe and employ aspects of the silhouette method [33] to identify imprint curves for five-axis motion by using pseudo-inserts of a fillet-end mill. The pseudo-inserts are introduced to calculate the approximate direction of motion for points on the cutter. A point P on each pseudo-insert is on an imprint if the moving direction of P lies in the tangent plane of the torus at P. They approximate this tangency condition with the cross product method, saying that the imprint curve can be obtained from the cross product of the normal vector to the plane of the insert and the local direction vector of the center point of the insert. Important to note, however, is that the instantaneous velocity vector should have been used instead of the approximate local direction vector to obtain the exact points on imprint curves. Furthermore, the approach is applicable only to the toroidal bottom and not to the side of the cutter and inductively validated by experiments and simulations.

Mann and Bedi [34] generalize the cross product imprint method for the toroidal cutter of Roth et al. [32] to five-axis motion of the cutter which is a body of revolution. They explain how the cross product method produces graze points with an alternative interpretation of the tangency condition (Eq. (1)): For a surface of revolution S, a compact sphere tangentially contacting the rotational surface is investigated, whose center is O and radius