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Computerized simulation of tooth contact and error sensitivity investigation for ease-off hourglass worm drives

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ABSTRACT

In the present paper, the tooth contact characteristics and the error sensitivity of the ease-off dual-torus double-enveloping hourglass worm drive (the DTT worm drive) are considered. The ease-off hourglass worm drive is composed of a standard hourglass worm and a modified worm wheel. In order to determine the instantaneous contact point of the mismatched worm pair, the two-stage downhill secant method (the TSDS method) is proposed to be used for solving the nonlinear contact equations. Roughly speaking, the main merits of the TSDS method are lower sensitivity to the initial guess, without a request for computing the Jacobi matrix and no singularity near the genuine solution. After ascertaining the transient contact point, the momentary contact ellipse, the instantaneous transmission ratio and some other tooth contact features are worked out by means of an improved local synthesis approach, which is proposed for point contact gear drives in this article. The results of computerized simulation show that, the mismatched worm gear set is able to accomplish desirable point contact, and has respectable load carrying capacity. On the other hand, the mismatched worm gearing is insensitive to the alignment errors, and its property of the transmission error is advantageous to reduce the noise and vibration.

1. Introduction

Generally speaking, the double-enveloping hourglass worm gearing has high load carrying capacity because it is capable of implementing multi-tooth double-line contact. However, the corresponding adverse effect is that the globoidal worm drive is extremely sensitive to manufacturing errors, misalignment, and load-dependent and heat-dependent deformations. In terms of a toroidal worm gear set, these errors and deformations can easily cause the teeth edge contact and the tooth surfaces curvature interference, and even lead to the early failure of the drive. One of the approaches to solve this problem is to substitute the line contact between the worm pair by the point contact by accomplishing the ease-off flank modification.

In order to study the attributes of the mismatched toroidal worm pair, the tooth contact analysis (TCA) technique needs to be adopted. This basic concept of the TCA methodology was originally proposed in the early 1960s and then was applied to the spiral bevel and hypoid gear drives successfully [1]. Afterward, some researchers investigated the cylindrical worm drive with a localized bearing contact by means of the TCA methodology. According to the definition in the DIN standards, the concerned investigations involved the ZI worm drive [2], the ZK worm drive [3–8], the ZC worm drive [9] and the ZN worm drive [10].

Some studies were made even on the localizations of the bearing contact and the transmission error functions of the spiroid worm drive and the face-worm gear with cylindrical worm also by means of the TCA methodology [11–14].

The basic and core work of the TCA procedure is to determine the instantaneous contact points between the tooth surfaces of the point contact gear drive by solving a system of nonlinear equations in several variables. Specifically for the double-enveloping worm pair, the worm surface is generally represented by three related parameters, as well as the tooth surface of the mating worm gear is represented by four related parameters. The simulation of the contact of the worm gearing, thereby, requires the solution of eight nonlinear equations, by selecting one parameter to be the input. This is frequently a difficult task, so that there is little information in the literature about the concrete procedure of how to attain the numerical solution of the above system of nonlinear equations, as far as the author has known. Although it was recommended to solve the nonlinear equations by utilizing the Newton–Raphson method [13], the substantial difficulties exist yet in the practical computation process, which can be noted in brief as follows.

The first is that guess value for the solution of the nonlinear equations has to be sufficiently close to the solution. Otherwise, the convergence of the iteration process cannot be guaranteed.

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Nomenclature

a	Center distance of the worm pair
a_e, b_e	Lengths of the semi major axis and the semi minor axis of the momentary contact ellipse, respectively
a_{ea}	Distance from the center of the top circle arc of the worm wheel to the center line of the worm wheel
$a_{l,l-2}$	Technical center distance during machining the worm ($l = 3$) or the toroidal hob ($l = 6$)
$\Delta a_{l,l-2}$	Modification quantity of the center distance during machining the worm ($l = 3$) or the toroidal hob ($l = 6$)
$\Delta a, \Delta\Sigma, \Delta b, \Delta c$	Alignment errors of the worm drive
$\boldsymbol{g}_1^{(l)}, \boldsymbol{g}_2^{(l)}$	Two principal directions of generating flank Σ_l of grinding wheel l, $l = 3, 6$
$\boldsymbol{g}_1^{(12)}, \boldsymbol{g}_2^{(12)}$	Two relative principal directions
$\hat{h}$	Step length factor
$i_{l-2,l}$	Technical transmission ratio, $l = 3, 6$
i_{12}, i_{12}^*	Theoretical transmission ratio and instantaneous transmission ratio of the mismatched worm pair, respectively
$[\boldsymbol{J}_s]$	Quasi Jacobi matrix
$k_1^{(l)}, k_2^{(l)}$	Two principal curvatures of generating flank Σ_l of grinding wheel l, $l = 3, 6$
$k_\xi^{(l)}, k_\eta^{(l)}, \tau_\xi^{(l)}$	Curvature parameters of the worm helicoid ($l = 1$), the worm gear tooth flank ($l = 2$), and the hob generating flank ($l = 4$)
$k_1^{(12)}, k_2^{(12)}$	Two relative principal curvatures
$\boldsymbol{n}$	Unit normal vector
$\boldsymbol{N}_{l,l-2}$	Normal vector of the instantaneous contact line, $l = 3, 4, 6$
p_l, q_l	Center coordinates of the working arc profile of grinding wheel l in its axial section, $l = 3, 6$
$\boldsymbol{r}$	Radius vector
r_{dl}	Radius of grinding wheel l, $l = 3, 6$
r_{f1}	Root radius of a toroidal worm at its throat
R_{a1}, R_{a2}	Radii of the top circle arcs of the worm and the worm wheel, respectively
$R[\boldsymbol{i}, \varphi]$	Rotation transformation matrix around $\boldsymbol{i}$ axis
$R[\boldsymbol{k}, \varphi]$	Rotation transformation matrix around $\boldsymbol{k}$ axis
$\boldsymbol{u}$	8- or 9-dimensional column vector
$\boldsymbol{V}_{l,l-2}$	Relative velocity vector, $l = 3, 4, 6$
$\boldsymbol{V}_{12}$	Relative velocity vector of the mismatched worm pair
w_s	Downhill factor
$\boldsymbol{\alpha}_m$	Two orthogonal unit vectors, $m = 1, 2$
β_l	Setting angle of the axis of grinding wheel l, $l = 3, 6$
$\Delta\varphi_2^*$	Rotation angle error of the worm wheel
ρ_l	Radius of the working arc profile of grinding wheel l in its axial section, $l = 3, 6$
ϕ_l, θ_l	Two parameters of the generating flank Σ_l of grinding wheel l, $l = 3, 6$
φ_l	Rotation angles of cutter frame l, which holds the grinding wheel l, $l = 3, 6$
φ_{l-2}	Rotation angle of the blank of the worm ($l = 3$) or the toroidal hob ($l = 6$)
$\varphi_4^*, \varphi_1^*, \varphi_2^*$	Rotation angles of the hob, the worm and the worm gear, respectively
$\boldsymbol{\Phi}^{(l,l-2)}$	Meshing function, $l = 3, 4, 6$
$\boldsymbol{\Phi}^{(12)}$	Meshing function of the mismatched worm pair
$\boldsymbol{\omega}_{l,l-2}$	Relative angular velocity vector, $l = 3, 4, 6$

Secondly, the Jacobi matrix needs to be worked out artificially. In fact, this is costly because every component equation is quite complicated.

As the final point, the Jacobi matrix may be singular in the process of approaching the solution of the nonlinear equations. In such case, the iteration cannot go on smoothly any more, and of course, the solution cannot be gotten.

With the purpose of overcoming these obstacles, some researchers suggested a new idea to ascertain the momentary contact point by calculating the minimum clearance between the point contact tooth surfaces [15]. Nevertheless, the computational practices show that this algorithm will cause a sizable amount of calculation. Recently, Shi et al. studied the bearing contact stabilization of the plane double-enveloping toroidal worm gear set by means of the finite element method [16].

In this paper, an effective solution for the preceding contact equations is proposed. The previous local synthesis method for point contact gear drives is improved substantially. The meshing features of the point contact double-enveloping hourglass worm pair are investigated elaborately, by taking the dual-torus double-enveloping hourglass worm drive (the DTT worm drive) as an example. The helicoidal surface of the DTT worm can be ground by using a grinding disk according to its formation mechanism. The used disk-shaped grinding wheel has two generating tori, which are symmetrical about the mid-plane of the grinding wheel. When the mismatch modification does not bring into effect, the DTT worm wheel should be enveloped by a toroidal hob cutter, whose generating surface has to be severely identical with the helicoid of the mating DTT worm. In such case, the line contact can be performed between the obtained DTT worm gearing [17].

Up to now, the meshing performance and lubrication behavior of the full conjugate DTT worm drive have been investigated [18–21]. As the manufacture mechanism of the point contact DTT worm gearing is different from that of the line contact one, it is well worth being explained as below.

In the process of producing the point-contact worm pair, the worm helicoid, Σ_1, is enveloped by the generating surface Σ_3 of grinding wheel 3, while the generating surface Σ_4 of the toroidal hob cutter 4 is enveloped by the generating surface Σ_6 of grinding wheel 6. The worm gear tooth surface, Σ_2, is generated by the generating surface Σ_4 of the toroidal hob cutter 4. At the end, let the worm helicoid, Σ_1, be in mesh with the worm gear tooth surface, Σ_2, in accordance with the requirement of the design drawing, and then the worm pair with localized contact can be obtained.

The localized bearing contact between the surfaces, Σ_1 and Σ_2, can be realized owing to the difference between the surfaces Σ_1 and Σ_4. Certainly, this difference should not be great. Naturally, the difference between the surfaces Σ_1 and Σ_4 also requires the deviation between the surfaces Σ_3 and Σ_6.

2. Cutting meshings of the worm and worm gear

2.1. Characteristic parameters for generating surface Σ_l

The right-handed spiral coordinate system $\sigma_{al}(o_{al}; \boldsymbol{i}_{al}, \boldsymbol{j}_{al}, \boldsymbol{k}_{al})$, $l = 3, 6$, is rigidly associated with the grinding wheel l as illustrated in Fig. 1. When $l = 3$, it corresponds to the cutting meshing of the toroidal worm. When $l = 6$, it corresponds to the cutting meshing of the toroidal hob. This agreement is also effective hereinafter. The axial section of the disk-shaped grinding wheel l intersects with its generating torus along the working arc profile. By means of the circle and sphere vector functions [22], the vector equation of the generating flank Σ_l of the grinding wheel l