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Industrial design using interpolatory discrete developable surfaces

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A B S T R A C T

Design using free-form developable surfaces plays an important role in the manufacturing industry. Currently most commercial systems can only support converting free-form surfaces into approximate developable surfaces. Direct design using developable surfaces by interpolating descriptive curves is much desired in industry. In this paper, by enforcing a propagation scheme and observing its nesting and recursive nature, a dynamic programming method is proposed for the design task of interpolating 3D boundary curves with a discrete developable surface. By using dynamic programming, the interpolatory discrete developable surface is obtained by globally optimizing an objective that minimizes tangent plane variations over a boundary triangulation. The proposed method is simple and effective when used in industry. Experimental results are presented that demonstrate its practicality and efficiency in industrial design.

1. Introduction

A surface $\mathcal{S}$ is ruled if through every point on $\mathcal{S}$ there is a straight line (called a ruling) lying in $\mathcal{S}$. If at each ruling the tangent planes to $\mathcal{S}$ remain unchanged, $\mathcal{S}$ is a developable surface. Developable surfaces can be flattened into a plane without stretching and distortions [1]. This unique property makes developable surfaces much desired in many design tasks in the manufacturing industry, including the design of aircraft, ship hulls, automobiles and garments [2], using the inelastic material of sheet metal, plywood, cardboard and cloth, etc.

Most commonly used free-form surfaces in industry are doubly curved [3]. So special efforts have to be paid to design with developable surfaces. The first computer treatments of developable surfaces were given in [4,5], in which two classes of developable surfaces were considered. The first is constructed with a given directrix and a given generator direction, and the second is constructed by interpolating two boundary curves. Developable free-form surfaces, in terms of Bezier surfaces, were studied in [6,7] and later were extended to B-spline form in [8]. However, nonlinear characterizing equations needed to be solved for satisfying the developability conditions on these developable free-form surfaces. A novel work was presented in [9] that can generate developable Bezier surfaces through a Bezier curve of arbitrary degrees, without solving nonlinear characterizing equations. Aumann's developable Bezier surface [9] was extended to be compatible with B-spline control nets in [10].

Another distinct direction of designing with developable Bezier and B-spline surfaces was to use the dual space from the point of view of projective geometry. In this perspective, a developable surface is treated as the envelope of a one-parameter family of tangent planes, and thus can be represented by a curve in a dual projective 3-space. This direction was typified in the work [11–13]. If sufficient differentiability is assumed, developable surfaces can only be part of plane, cone, cylinder, tangent surface $\mathcal{S}(t, r) = c(t) + rc'(t)$ of a twisted curve $c(t)$ or a composition of them. Design of smooth developable surfaces using the above analytic methods often exhibits inflexibility behavior, i.e., very few degrees of freedom can be provided.

Towards a flexible and efficient tool in industrial design, mesh discretization of smooth developable surfaces has received considerable attention recently. Both quadrilateral and triangular meshes were adopted. Liu et al. [14] showed that the quadrilateral meshes with planar facets (called *PQ meshes*) are particularly suitable for free-form glass structures in architectural design. The relation between PQ meshes and conjugate curve networks was also given in [14]. For the discrete-differential-geometry properties of PQ meshes, a rigorous analysis was presented in [15]. Constrained triangular meshes are also studied in modeling and tessellating arbitrary free-form developable shapes [16–22]. The condition of a triangulation approximating a developable surface was given in [16]. A nice property of these mesh discretizations is that both smooth and buckled developable patches joined along crease lines can be modeled in a unified framework [15,23,24].

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In this paper, we adopt a constrained triangulation as a discrete developable surface. We formulate the constraints into optimization criteria and present a simple dynamic programming solution for finding such a mesh surface that achieves globally maximized developability. A distinct characteristic of our method is its compatibility with a user-friendly interactive design interface with sketching. Compared to previous strip-based triangulation methods [16,18,20], the presented method constructs developable meshes interpolating closed 3D curves which can enclose a multiple-connected region when projected back to the sketching plane.

2. Related work

Design directly using developable surfaces is desired in many industrial applications. Many different design styles have been proposed. In [7], to satisfy the nonlinear constraints, a strict and complicated process is required to set up the positions of control points of a developable Bezier surface. A better interaction process was proposed in [9], in which the user first draws a Bezier curve c_1 of arbitrary shape and degree, and then a developable Bezier surface is constructed by interpolating c_1 and some points on the other boundary curve c_2. This method however had the limitation that only five degrees of freedoms can be used to specify the point positions on the curve c_2. Design of a developable surface as a 3D curve in dual space offers a new perspective [11–13], but the user usually loses some intuitions when interactively designing in a projective dual space. A novel interactive style was presented in [25] with which the user can control a geodesic to efficiently animate a paper bending. The primitive shape represented in [25] is however restricted to a rectangle strip.

Design using discrete mesh surfaces widely enlarges the variety of surface shapes that can be represented by developable meshes. Rectangle-like developable triangle strips interpolating two arbitrary boundary curves were studied in [18,20]. The Hamilton principle was introduced in [24] to animate an inelastic disk-like sheet. With an optimized segmentation, developable triangle strips can be used to model complicated 3D graphics models [21]. An elegant optimization process was proposed in [14] for perturbing a quad mesh into a PQ mesh so that free-form shapes, especially architectural structures, can be represented. The PQ meshes were extended in [15] to model free-form developable surfaces with curved folds. The developable meshes generated by the method in this paper interpolates closed 3D curves and can model free-form shapes in an intuitive and efficient way.

Paper-and-pencil-based sketches are suited well to the traditional design habits of human beings. Many sketch-recognition systems have been proposed to support pen-input interaction concerning general 2D geometry constructions [26]. Inspired by the user-interface given in [19,27], in this paper we also develop a sketch-based interactive design interface: The user sketches arbitrary closed curves in a sketch plane and manipulates points' normals to generate 3D boundary curves. In this work, a dynamic programming solution is presented to efficiently generate a developable mesh surface interpolating the given arbitrary 3D boundary curves, whose projection into the sketch plane can bound a multi-connected region. Dynamic programming has been used in [18,22] for modeling developable meshes in rectangle-like strips. Since any triangle strip is trivially discrete developable, compared to the work in [18,22], in this paper we use an elegant measure of discrete developability and the major contribution is that by optimization using the measure of developability, the free-form developable shape that we can model is extended from simple rectangular strips [18,20,22] to branched shape (Fig. 10(a)) and shape with inner holes (Figs. 12 and 17).

3. Overview of the design system

The goal of the proposed system is to provide a simple and easy-to-use software tool with which common users can design free-form developable surfaces for diverse applications. The system interface (Fig. 1 left) explores a point in the tradeoff between expressiveness and naturalness. The user sketches (Fig. 1 right) arbitrary closed curves that are shown in the left panel of the interface. The planar sketching curves are obtained by tracing pen/mouse moving. The system reports the positions of pen/mouse motion using a polyline with a large number of vertices. The polyline is simplified using the classic Douglas–Peucker algorithm [28]. The simplified polyline is the initial draft capturing the design intention of the user. As it is often desired that the initial draft can be modified and refined by continuous interaction [29], the polyline is approximated by a B-spline curve of degree three. The approximation error is globally controlled within a tolerance $L/100$, where L is the length of the polyline. Non-professional users often prefer to modify the curve shape by directly moving curve-points (called *handle points* below). Advanced B-spline techniques [30] are provided in the system so that the user can modify the positions of handle points on the curve, add or delete some handle points on the curve, all through simple sketching operations [27]. One or more non-intersection closed curves representing a multi-connected region can be sketched in this phase.

The right panel of the system (left in Fig. 1) contains an OpenGL window that renders the closed planar curves in $\mathbb{R}^3$. The user can modify the positions of handle points in this window. But the movement of handle points is restricted in the normal direction of the sketching plane in the left panel. In this way the projection of 3D curves (in the right panel) onto the sketching plane is guaranteed to be the sketched 2D planar curves (in the left panel). The interactive design of closed 3D curves is continuous in both left and right panels.

Given a set of closed 3D curves as boundaries, an interpolatory discrete developable surface is generated using the method proposed in the next section. The user can browse the results in different rendering modes (shading, wireframe and others) also in the right panel. Since the proposed developable surface interpolation method is fast, the user can interactively modify the shape of boundary curves and re-generate the interpolating surface in real-time. User experiences in Section 5 show that great productivity is achieved by this interactive manner and diverse free-form shapes can be designed directly using discrete developable surfaces.

4. Interpolatory discrete developable surface

In this section we present a special type of triangulation as a discrete developable surface and impose a measure of discrete developability on it. Firstly the 3D curves sketched in the system are discretized into polylines P using the solution to a $Min - \#$ problem [18]. Given polylines P with edges E_P and vertices V_P, a *boundary triangulation* of P is defined as a triangulation T of V_P that satisfies the following conditions:

1. $E_P \subseteq E_T$ and $V_P = V_T$, where E_T and V_T are the edge set and vertex set of T;
2. For each $v \in V_T$, the local surrounding area of v in T is homeomorphic to a two-dimensional half space $H^2 = \{(x_1, x_2) \in \mathbb{R}^2, x_1 \geq 0\}$;
3. If an edge $e \in T$ is incident to only one face, then $e \in E_P$.

Some notations used in this paper are summarized in Table 1.

Observing the fact that along each ruling of a developable surface, the tangent plane does not change, we define the following objective function with which the optimal boundary triangulation maximizes a discrete developability measure.