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A new calculation method for the worst case tolerance analysis and synthesis in stack-type assemblies

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ABSTRACT

Productivity and industrial product quality improvements entail a rational tolerancing process to be applied as early as product design. Once functional conditions are defined, an optimal specification for each component in a mechanical system is to be developed. Despite numerous studies in this area, the problem is still far from solved. It may be decomposed into two stages: development of specifications based on standards, or qualitative synthesis, and calculation of tolerances. To the extent that these two sets of problems are related, we propose to address them in parallel. In this paper, we present an original method that enables us to solve these two problems for the case of serial assembly (stacking) without clearances. This method is based on the use of influence coefficients to obtain the relationship between the functional tolerance and the tolerances associated with the geometry of the mechanism's interface surfaces. We will describe a calculation algorithm that helps obtain influence coefficients solely from the assembly's geometric definition. Then, we will show that under our working hypothesis, this relationship is piecewise linear.

1. Introduction

Current methods of 3D tolerance analysis that are used in Computer Aided Tolerancing software mostly rely on parameterized geometry. Such geometry is defined in 3D by way of a variational or parametric CAO model. In general, the objective then is to determine the relationships between variations in parameters used to describe the geometry and variations in the variables associated with the functional conditions. A functional condition is most often described by limits imposed on variables as a result of parameters, such as clearance or distance between two points on two functional surfaces. A component specification with tolerances compliant with ISO or ASME standards assures a large number of functionalities but does not translate into tolerances that directly affect design parameters. The location of a hole, for instance, may be defined in Cartesian coordinates with respect to two orthogonal planes and using two dimensional parameters but a functional tolerance on the position of the hole axis may define a cylindrical tolerance zone which cannot be described by independent bounds on the two parameters.

In this paper, we propose a model and a method which help in the case of component stacking analytically to get the relationships between the variables that define functional conditions and the values of tolerances on position and orientation to obtain the best functional fit and conformance to standards. A synthesis of tolerances consists in developing specifications best suited to functional requirements. Our method compares specifications of various types and helps select the most appropriate ones but also to allocate the values of tolerances to the assembly's components. This problem arises for the designer at the preliminary design phase where it is possible to study the effects of the selected geometry on the accuracy required to satisfy geometric conditions as well as during detail design where necessary and sufficient standard specifications have to be stated on the final drawings.

We propose an original method of calculation to obtain the relationships connecting tolerance t_{C_f} on the functional condition to tolerances t_{ps} associated with surfaces s of mechanical components p.

Thus, we propose a new calculation algorithm helping express the relationship in linear form (Eq. (1)) wherein c_{ps} are referred to as "influence coefficients":

$$t_{cf} = \sum c_{ps}.t_{ps}. \quad (1)$$

This linear relationship is well known for unidirectional dimension chains. Anselmetti notes that in three dimensions a study of simple examples helps obtain this relation but acknowledges the difficulty for the more general cases [1]. A more complex example is described by Jian et al. wherein the influence coefficients are calculated analytically [2]. But as far as we know, the exact conditions for which this linear relationship is valid or a method to get the coefficients in the general case have never been presented.

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We will show that the value of influence coefficients c_{ps} depends in the general case on the selected allocation of tolerances. In fact, this relationship is piecewise linear (in the research hypotheses to be defined below).

The three groups of tolerancing problems listed by Salomons [3] – specification, tolerance analysis, and tolerance synthesis – are closely related, and synthesis cannot be handled in isolation from the other two. That is why we propose to define an approach that will allow qualitative tolerancing (development of specifications and reference systems) and quantitative tolerancing (values of tolerances) to be performed simultaneously. Thus, this method will help optimize the overall tolerancing of a mechanism from the very start of the digital process.

Following a review of literature, we will examine the case of a component stack with no-clearance contact, and then look at the effect of possible clearances at the interfaces. The case of parallel linkages will not be presented in this paper but the approach can be generalized.

2. Previous research

Most works on the synthesis of tolerances agree that there are two phases. The objective of the first phase is to obtain the equations that relate geometric deviations to variations in functional conditions. We will refer to these as tolerancing equations. The second phase follows one of two possible paths: it either looks for the worst case or uses a statistical approach. The worst-case method would have to take into consideration the inequalities related to geometric deviations bounded by tolerance. Under the statistical method, statistical data related to deviation variables will be assumed as known and will be used to get statistics on variables that describe the functional conditions.

2.1. Deriving tolerancing equations

One of the methods requires that relative positions be modeled by transformation matrices and that an assembly loop, then, be described by a product of these matrices. By expanding this equation into a first-order Taylor series, we obtain the linear relations between variations in functional conditions and variations in the different variables that describe the geometry and the assembly [4]. In the direct linearization (DLM) method presented by Chase, all linear and angular geometric variations of contact surfaces can be taken into consideration [5].

Franciosa, for his part, proposes a numerical procedure to simulate assembly constraints using the formal tool of linearized transformation matrices [6].

These equations can also be obtained by using torsors. Ballot and Bourdet define two types of torsors [7]: deviation torsors that describe variations in geometry and linkage torsors that describe the small displacements allowed by linkages with or without clearances. For each linkage, the relation between surface displacement torsors is written as follows. In the mechanism's global reference frame R with surface S_a belonging to part P_1 and forming a linkage with surface S_b of part P_2, one would write (Eq. (2)):

$$\{T_{S_a/S_b}\} = (\{T_{S_a/P_1}\} + \{T_{P_1/R}\}) - (\{T_{S_b/P_2}\} + \{T_{P_2/R}\}). \quad (2)$$

By removing the indeterminate variables which correspond to the degrees of freedom of the linkages in question, we get compatibility equations for every linkage that makes up the mechanism. From this, a linear relationship is obtained associating small displacement torsor components for each interface surface to those of a terminal surface to which the functional condition is applied [8].

This same formal tool is used in manufacturing to determine relative deviations in component surfaces based on the knowledge of the manufacturing process and the deviations specific to each machining operation and to each mount [9].

The advantage of the method is that it automatically eliminates variables that represent linkage degrees of freedom whereas other methods perform this elimination implicitly through appropriate parameter selection.

Laperriere describes geometric variations through components of small displacement torsors in a single reference frame. A kinematic loop closure equation is a relationship between its torsors. To get scalar relations (6 scalar equations per loop), the torsors have to be expressed in terms of the same point and projected onto the same vector basis. This model is referred to as a Jacobian-torsor model [10].

2.2. Tolerancing inequalities

Tolerancing equations, whether obtained by direct linearization or from torsors, can be used for statistical analysis. It is assumed that most frequently geometric variations and displacements in the linkages are independent random variables. The statistics for the resultant variable characterizing the functional condition are then computed analytically or through the Monte Carlo method using hypotheses regarding each variable's distribution function [4,8]. In reality, clearance configurations are frequently unknown, clearances are a positive for the assembly, and, consequently, do not play the same role as geometric variations which are a negative for the assembly. This is demonstrated by Dantan in [11]. On the other hand, these equations do not show the relationships between geometric and functional tolerances.

Therefore, contact conditions within the linkages and tolerances should be modeled using deviation inequalities. Assembly conditions will be verified in the worst case if the system of equations and inequalities is compatible. Laperriere and Desrochers model tolerances by bounding deviation torsor components. The Jacobian-torsor model provides them with a linear relation between deviation components. The effect of each tolerance on compliance with the functional condition may then be expressed aiding the designer in optimizing his or her tolerancing [12]. The modeling method proposed by Teissandier et al. is very close to this model [13]. However this method does not account for the cross-coupling between small displacement components for a given standard tolerancing and, hence, runs the risk of producing excessive quality. In fact, the maximum values of small displacement components assuring compliance with the geometric specifications are not independent.

Petit and Giordano model ISO tolerances using inequalities with small displacement torsor components and provide a geometric representation of each tolerance zone in the form of a convex envelope or a polytope referred to as the deviation domain [14]. Then, deviation components are represented by deviation domains whereas contact conditions are modeled by clearance domains [15]. Davidson uses a different formal method but obtains the same convex envelopes that he refers to as T-maps [16]. The difference between the two models is formal. Whereas the domains exist in the torsor space, the T-maps evolve in the parametric space. The former is a vector space with a dimension of at most 6, and geometric property deviations are modeled by parameterizing the domains. The T-map space, on the other hand, may include dimensional components to handle maximum material, for instance [17]. In both the cases, worst-case tolerance stack-up will be modeled by the *Minkowski* sum of tolerances. When the number of parts is large and the shape of the tolerance zone complex, the computation of the Minkowski sums presents a problem because of combinatorial explosion. Using the T-map model, Jian et al. use an example to get the analytical relationship between the tolerances in the form of Eq. (1).