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Computing point-set surfaces with controlled spatial variation of residuals

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ABSTRACT

This paper presents an accurate method for computing point-set surfaces from input data that can suppress the noise effect in the resulting point-set surface. This is accomplished by controlling spatial variation of residual errors between the input data and the resulting point-set surface and offsetting any systematic bias. More specifically, this method first reduces random noise of input data based on spatial autocorrelation statistics: the statistics Z via Moran's I. The bandwidth of the surface is adjusted until the surface reaches desired value of the statistics Z corresponding to a given significance level. The method then compensates for potential systematic bias of the resultant surface by offsetting along computed normal vectors. Computational experiments on various point sets demonstrate that the method leads to an accurate surface with controlled spatial variation of residuals and reduced systematic bias.

1. Introduction

This paper presents an accurate method for computing point-set surfaces from input data that can suppress the noise effect in the resulting point-set surfaces. Point-set surfaces (PSSs) are continuous surfaces defined directly from point sets. Since its original inception [1–3], the PSS and its many variants have been widely used in various graphics, visualization, geometric modeling and engineering applications. For example, we have applied the PSS in the context of computer-aided design and manufacturing and developed a new approach termed *direct digital design and manufacturing* [4]. This approach can enable direct digital design and manufacturing from massive scanned data, by passing the usual CAD model reconstruction. This includes the use of PSSs for direct rapid prototyping [5], NC machining [6], and direct Boolean intersection between CAD geometry and acquired geometry [7]. The projection operation in defining PSSs can also be applied for drawing curves onto digital surfaces in point-based modeling [8–10].

However, despite the broad usage of PSSs and its wide variants (for example, ψ-type M-Estimators [11] and the forward search algorithm [12] have been used to improve the shape quality of the resulting PSSs), thus far there has been limited work on examining the spatial distribution of residual errors of the resulting PSSs.

Our work in this paper assumes that the input points are points sampled from object surfaces contaminated with random noise. The premise of this work is the observation that, when the residual errors of input points with respect to the reconstructed PSS approach spatially random, the corresponding PSS approaches the true surface. Our approach is based on Levin's PSS [1]. We obtain the resulting PSS in a two-phase approach. We first vary the bandwidth of the PSS until the statistics Z and Moran's I (measures used for characterizing the spatial randomness of residual errors between the input data and the resultant PSS, which are introduced in Section 3) reach a specified random level. We then compensate for potential systematic bias by offsetting the points in the amount of mean error along the computed corresponding normal vectors.

Moran's I and corresponding statistics Z have been used in three dimensional coordinate metrology to examine randomness of geometric errors of B-spline surfaces [13]. However, in this paper, we reveal that controlling the randomness alone does not preclude systematic bias (as shown in Section 4.3). This observation has led to the second phase (the offsetting operation) in our method.

Various computational experiments demonstrate that, through controlled spatial variation of residual errors and the offset of systematic bias of the mean error, accurate PSSs can be obtained for input points sampled from various freeform shapes.

The remainder of the paper is organized as follows. Section 2 reviews related work on PSSs. Section 3 introduces Moran's I and corresponding statistics Z to measure the randomness of spatial patterns. Section 4 presents a method to compute the randomness of residuals in the reconstructed PSS and describes how to offset the potential bias in the surface. Section 5 details the proposed point-set surface reconstruction method. Experimental results are given in Section 6. Finally, conclusions are given in Section 7.

2. Related work

A point-set surface is a continuous surface defined directly from a point set. Given a point set $P = \{\boldsymbol{p}_i,\ i = 1, \ldots, n\}$, the original

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PSS is defined in two steps [1]: an MLS (moving least squares) projection procedure and a polynomial fitting procedure. First a local plane $H = \{\boldsymbol{x} | \boldsymbol{n} \cdot \boldsymbol{x} - D = 0, x \in R^3\}$ is found by minimizing the weighted sum of squared distances

$$\sum_{i=1}^{n} (\boldsymbol{p}_i \cdot \boldsymbol{n} - D)^2 \theta(\boldsymbol{q}, \boldsymbol{p}_i),$$

where $\boldsymbol{q}$ is the projection of $\boldsymbol{x}$ onto H, $\theta(\boldsymbol{q}, \boldsymbol{p}_i)$ is usually a Gaussian weight function with bandwidth h:

$$\theta(\boldsymbol{x}, \boldsymbol{p}_i) = \mathrm{e}^{-\frac{\|\boldsymbol{x}-\boldsymbol{p}_i\|^2}{h^2}}. \tag{1}$$

After H is found, the second step finds a local polynomial by taking the H as the reference plane and using a similar weighted least squares method.

Amenta and Kil [3] generalized the MLS projection procedure via the concept of extremal surfaces. The resultant surface of the MLS projection is represented by an implicit function which is the product of a vector field $\boldsymbol{n}$ and the gradient of an energy function

$$e(\boldsymbol{x}, \boldsymbol{n}(\boldsymbol{x})) = \sum_{i=1}^{n} ((\boldsymbol{x} - \boldsymbol{p}_i)^T \boldsymbol{n}(\boldsymbol{x}))^2 \theta(\boldsymbol{x}, \boldsymbol{p}_i).$$

Such a surface definition for the MLS surface is conducive to calculating surface characteristics such as curvatures [14].

Since the original definitions, many variants of PSSs have been developed. They differ, for example, in the strategies used to specify bandwidths (sample density [15], local feature size [16], curvatures [17]), in the surface models used to fit (planes [3], spheres [18], polynomials [1]), in the weight functions (isotropic and anisotropic weight functions [19], singular weight functions [20]), and in fitting criterion (least squares criterion, ψ-type M-Estimators [11], the maximum residual criterion [12]), and so on.

Using singular weight functions and a proper centroid function, PSSs can interpolate locations and derivatives at these locations [20]. The interpolatory PSS is suitable for noiseless data. Algebraic PSSs [18] avoid the polynomial fitting procedure by fitting spheres in the projection procedure. They use spheres instead of general polynomials because spheres are easy to fit and there is a close form of closest points on spheres. However, not all surfaces can be accurately approximated locally by a sphere.

The bandwidth h in weight functions is an important parameter for PSSs because a PSS with a larger bandwidth is smoother but may smooth out small or sharp features, while a PSS surface with smaller bandwidth is more faithful to the input data but may be rough. The bandwidth h is typically selected according to sample density of points, local feature size, curvatures, and so on.

Pauly [15] defined a bandwidth by the function $h_{\boldsymbol{x}} = c/\rho(\boldsymbol{x})$, where c is a fixed scale factor, $\rho(\boldsymbol{x}) : R^3 \to R^+$ is a continuous, smooth function approximating the local sampling density. The ρ is computed by first estimating the local sample density for each point $\boldsymbol{p}_i \in P$ by $\rho_i = k/r_i^2$ and then interpolating by standard scattered data approximation techniques, e.g. radial basis functions, where r_i is the minimum radius of the sphere centered at $\boldsymbol{p}_i$ and containing k nearest neighbors to $\boldsymbol{p}_i$.

Dey and Sun [16] take the bandwidth to be a fraction of the local feature size and define their PSS by the weight function

$$\theta(\boldsymbol{x}, \boldsymbol{p}) = \mathrm{e}^{\frac{-\sqrt{2}\|\boldsymbol{x}-\boldsymbol{p}\|^2}{\rho^2 l_{fs}(\hat{\boldsymbol{x}}) l_{fs}(\hat{\boldsymbol{p}})}},$$

where $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{p}}$ are closest points on the sampled surface S from points $\boldsymbol{x}$ and $\boldsymbol{p}$ respectively, and $|\rho| < 1$ is a scale factor. The local feature size $l_{fs}(\boldsymbol{x})$ at a point $\boldsymbol{x} \in S$ is defined as the distance from the point to the nearest point of the medial axis of S.

Wang et al. [17] used an optimal bandwidth in the second step of the definition of Levin's PSSs. They formulated the weighted least squares polynomial fitting by the kernel regression and found the optimal bandwidth by minimizing an approximated error evaluating the kernel regression performance. In their formula, the bandwidth is selected by combining noise level, sample density, and curvatures.

In addition to the isotropic weight function given in Eq. (1), an individual ellipsoidal weight function to each sample point is used to define the PSS in [19], which is given by

$$\omega_i(\boldsymbol{x}) = \theta(\|H_i^{-1}(\boldsymbol{x} - \boldsymbol{p}_i)\|),$$

where θ is a smooth monotonically decreasing function, H_i is an ellipsoid oriented so that one of its axes points into the normal direction and the other two align with the principal curvature directions. There also is the bandwidth selection problem for the anisotropic weight function.

In order to preserve small features and be less influenced by outliers, robust implicit PSSs are defined by combining implicit PSSs with robust local kernel regression [11]. Instead of the ordinary least squares criterion, ψ-type M-Estimators are used to assign outliers less weight, i.e. additional weight functions are used in the objective function. Besides the bandwidth used in the spatial weight function, two additional bandwidths σ_r and σ_n are introduced in the other two new weight functions. The σ_n is used in the weight function of differences between predicted gradients and sample normals. The σ_r is used in the weight function of residuals of values of implicit surface function. Smaller values of the σ_n lead to sharper results.

Based on the fact that sharp features are formed by multiple surfaces, a forward search method is introduced in [12] to find points of a smooth region by a maximum residual criterion. Sharp features are identified by intersections of surfaces. The final results are robust to outliers since the forward search method gets rid of outliers from the fitting procedure.

The moving least squares interpolation scheme is analyzed in [21], where a surface is reconstructed from point cloud data with normal vectors. The surface is defined by the implicit function

$$F(\boldsymbol{x}) = \frac{\sum_{i=1}^{n} \theta_i(\boldsymbol{x})((\boldsymbol{x} - \boldsymbol{p}_i)^T \boldsymbol{n}_i)}{\sum_{i=1}^{n} \theta_i(\boldsymbol{x})},$$

where $\theta_i(\boldsymbol{x})$ is a weight function and $\boldsymbol{n}_i$ is the normal vector at point $\boldsymbol{p}_i$. Under some assumptions about sample densities, local feature size, and the bandwidth of the weight function, the surface $F(\boldsymbol{x}) = 0$ will lie in neighborhoods of underlying surfaces. The sizes of the neighborhoods are bounded by a value comparable with the point space of the input point cloud. It can be proved that the projection procedure converges and the resultant surface is isotopic to the underlying surfaces [22].

In this paper, we follow the original definition of the point-set surface [1], but with the explicit goal of ensuring that the residual errors between the input points and the resulting surface are spatially independent and free from systematic bias. Although other filters such as bilateral filters on meshes [23] and feature sensitive filtering [24,25] exist, our approach differs in several aspects: (1) input model, our approach works directly on discrete unorganized points versus polygonal mesh; (2) output model, our approach generates a continuous, implicitly defined MLS surface, rather than another polygonal mesh; (3) noise characteristic, our approach explicitly quantifies the spatial correlation of the error distribution through Moran's I and statistics Z.