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Non-iterative approach for global mesh optimization

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Abstract

This paper presents a global optimization operator for arbitrary meshes. The global optimization operator is composed of two main terms, one part is the global Laplacian operator of the mesh which keeps the fairness and another is the constraint condition which reserves the fidelity to the mesh. The global optimization operator is formulated as a quadratic optimization problem, which is easily solved by solving a sparse linear system. Our global mesh optimization approach can be effectively used in at least three applications: smoothing the noisy mesh, improving the simplified mesh, and geometric modeling with subdivision-connectivity. Many experimental results are presented to show the applicability and flexibility of the approach.

Keywords: Mesh optimization; Non-iterative; Global Laplacian operator; Feature preserving

1. Introduction

Triangular meshes are commonly used as a representation of shape in many computer graphics applications. Many of these meshes are generated by implicit surface polygonization or by scanning devices. However, automatic meshing on free form geometry frequently produces meshes of unsatisfactory quality. As stability and convergence of various mesh processing applications depend on mesh quality, there is frequently a need to improve the quality of the mesh [1]. This improvement process is called *mesh optimization* [2]. It is useful to ease not only the display process, but also the numerical simulation, storing, editing, and animation [3]. Therefore, a lot of effort has been put into research on mesh optimization in the literature [4–7].

There is no precise definition of mesh optimization, since it often varies according to the targeted goal or application. One possible definition could be roughly stated as follows: Given a 3D triangular mesh, compute another triangular mesh, whose elements satisfy some quality requirements, while approximating the input well. There are numerous ways to measure the quality of mesh. General criteria include the size and shape of triangles and the valence of vertices. Often a combination of these criteria is desired in real applications. Some optimization techniques proceed by altering the input [1,8], and some generate a new mesh from scratch [4,9].

Considerable research has been conducted in the mesh optimization community by using local optimization approach. Local mesh optimization methods solve optimization problem in the vicinity of a specific mesh vertex and typically require the use of tools such as mesh modification and vertex repositioning. Mesh modification methods include edge swapping, vertex insertion (edge splitting, face splitting), vertex deletion (edge collapse) and local mesh retriangulation. Mesh modification methods change the topology of the mesh and therefore, may be more difficult to use in simulations requiring solution transfer from the original mesh to the improved mesh. Vertex repositioning methods are much related to mesh smoothing or denoising [1,10,11], where mesh vertices are moved one by one by performing some smoothing operators, such as Laplacian operator [1], in an iterative process.

In local mesh optimization, a local iterative procedure is used to update the positions of the vertices. The new position of a vertex may not solely depend on the set of old positions of its adjacent vertices but can depend on their previously calculated new positions, too. Hence, the result of one optimization pass

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through all vertices will depend on the order how the vertices are considered. Unfortunately, local optimization leads to a variety of artifacts such as geometric distortion and shrinkage due to the irregular connectivity of the mesh.

Global mesh optimization is oriented on optimization of mesh quality metrics for an entire domain and changes all the vertex locations in a mesh simultaneously [5,12,45]. Therefore research on efficient methods of global mesh optimization has become an important issue recently.

In [12], we proposed a global approach for surface smoothing with feature preserving. This paper extends the work of [12] and presents a non-iterative global procedure to improve the quality of triangular meshes by updating vertex positions while preserving the essential characteristics of the mesh surface and keeping the mesh close to the original one. We consider about the mesh optimization as a problem of finding an approximating surface with a global minimization of a surface fairing energy. Thus we adopt the Laplacian operator in a global way instead of a local way. The improved mesh is constructed by solving a spare linear system, which is fast and efficient in a linear running time, non-iteratively. Various geometric constraints are also considered in our approach to preserve the features of the original mesh.

Recently, a much similar approach for mesh optimization was proposed independently in [45]. The contributions of our global optimization for arbitrary meshes are summarized in the following:

- *Global operator:* The Laplacian operator is performed over the mesh but not over each vertex locally. The improved mesh is obtained by performing the global Laplacian operator.
- *Non-iterative:* Our global optimization approach is non-iterative. The improved mesh can be constructed by solving a sparse linear system.
- *Fast and efficient:* Our approach is fast and efficient as it only needs to solve a sparse linear system which can be effectively solved in a linear running time.
- *Feature preserving:* The improved mesh can keep the features of the original mesh without shrinkage and distortion by adding variety of geometric constraints in the linear system.

The rest of the paper is organized as follows. Section 2 lists some of the related work. Section 3 gives the mathematical formulization for global mesh optimization. A variety of linear vertex placement constraints are described in Section 4. Section 5 discusses some remarks on the approach. Additional experimental results are illustrated in Section 6. We conclude the paper in Section 7.

2. Related work

There are several related topics related to mesh optimization. One can refer to read some comprehensive papers on mesh optimization [2,4] and mesh smoothing [10,11] and the related applications including surface remeshing [13] and subdivision surfaces [14,15].

2.1. Mesh optimization

There are many optimization techniques for vertex repositioning. Most of them are based on the idea of local optimization and require an improvement of such mesh quality parameters as aspect ratio, area, etc. Hoppe et al. [2] described an energy minimization approach to solving the mesh optimization problem. The energy function consists of three terms: a distance energy that measures the closeness of fit, a representation energy that penalizes meshes with a large number of vertices, and a regularizing term that conceptually places springs of rest length zero on the edges of the mesh. Their minimization algorithm partitioned the problem into two nested subproblems: an inner continuous minimization and an outer discrete minimization. Turk [4] proposed an approach for distributing a given number of points over a mesh surface evenly. These points are connected to one another to create a re-tiling of a surface that is faithful to both the geometry and the topology of the original mesh surface. Therefore, these points will eventually become the vertices of the new mesh model. Recently, a procedure was presented to improve the quality of complex polygonal surface meshes without an underlying smooth surface using numerical optimization in [6]. The movement of the mesh vertices is driven by a nonlinear numerical optimization process. Knupp [16] studied three-dimensional unstructured tetrahedral and hexahedral finite element mesh optimization from a theoretical perspective.

2.2. Mesh smoothing

A great deal of smoothing techniques have been developed. Among the earliest of these methods is Laplacian smoothing [17] and its variations. Taubin [1] introduced signal processing on surfaces that is based on the definition of the Laplacian operator on meshes and developed a fast and simple iterative Laplacian smoothing scheme. Desbrun et al. [5] extended this approach to irregular meshes using a geometric flow analogy. Ohtake et al. [18] extended the Laplace smoothing by combining geometry smoothing with parameterization regularization.

Feature-preserving mesh smoothing methods [19–21] were mostly inspired by anisotropic diffusion in image processing [22]. These methods modified the diffusion equation to make it nonlinear or anisotropic, and thus could preserve sharp features.

Recently, Jones et al. [11] proposed a statistical method to anisotropically smooth a mesh in one pass. This approach predicts the location of a vertex based on its neighbors. Robust statistics are used to de-emphasize the contribution of vertices dissimilar to the one being predicted. Fleishman et al. [10] introduced a similar method based on bilateral filtering that is iterative. However, it is not straightforward to assign appropriate parameters to get good results in the algorithms.

2.3. Surface remeshing

Remeshing of surfaces has received considerable attention over the past few years. It also aims to create a new mesh with