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Robust parameter synthesis for planar higher pair mechanical systems

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Abstract

We present a parameter synthesis algorithm for planar, higher pair mechanical systems. The input is a parametric model of a mechanical system (part shapes and configurations) with nominal values and tolerance intervals for the parameters. The output is revised parameter ranges that guarantee correct kinematic function for all system variations. Nominal values are changed when possible and tolerance intervals are shrunk as a last resort. The algorithm consists of a three-step cycle that detects and eliminates system variations with incorrect kinematic function. The first step finds points in parameter space whose kinematic variation is maximal. The maximum of the higher pairs are derived by contact zone construction and are then combined into system maximums. The second step tests the points for correct kinematic function using configuration space matching and kinematic simulation. The third step adjusts the parameter ranges to exclude the points that fail the test. The cycle repeats until every point exhibits correct function. We demonstrate the algorithm on five real-world examples.

Keywords: Parameter synthesis; Higher pair; Kinematics; Configuration space

1. Introduction

We present a parameter synthesis algorithm for planar, higher pair mechanical systems. Parameter synthesis is a central part of kinematic synthesis, which is the task of designing a mechanical system that performs a specified kinematic function. Kinematic synthesis is an iterative process in which designers select a design concept, construct a parametric model, and assign parameter ranges in the form of nominal values plus tolerance intervals. The goal of parameter synthesis is to derive parameter ranges that guarantee correct kinematic function at a minimal cost. Overly tight tolerances can necessitate expensive manufacturing processes, whereas overly loose tolerances can lead to unreliable products.

The kinematic function of a system is the coupling between its part motions due to contacts between pairs of parts. A lower pair has a fixed coupling that can be modeled as a permanent contact between two surfaces. For example, a revolute pair is modeled as a cylinder that rotates in a cylindrical hole. A higher pair imposes multiple couplings due to contacts between pairs of part features. For example, gear teeth consist of involute patches whose contacts change as the gears rotate. The system transforms driving motions into outputs via sequences of part contacts.

Manufacturing variation causes the actual system to deviate from the nominal design, which causes incorrect kinematic function. One type of incorrect function is excessive deviation from the nominal part motion, such as a cam/follower that deviates by 5% from its prescribed path. A second type is a failure mode due to an unintended part contact, such as gears that jam because two pairs of teeth drive in opposite directions. System variation is modeled by generalizing the nominal design to a parametric family of designs. The part shapes and configurations are given in terms of symbolic parameters, such as lengths and angles, whose nominal values specify the nominal system. The allowable manufacturing variation is specified as tolerance intervals around the nominal parameter values. The parameter synthesis task is to assign parameter ranges that prevent incorrect kinematic function.

We formulate parameter synthesis as parameter space search [1]. The parameter space is an n-dimensional Euclidean space whose ith axis measures the ith design parameter. A point in parameter space represents a design instance. The parameter ranges define a hyper-box in parameter space that is centered at the nominal instance and that contains the allowable system variations. The synthesis task is to construct the largest possible hyper-box that is free of failure instances. We employ

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the *robust design* strategy [2,3] of first varying nominal values, which does not increase cost, and of tightening tolerances only as a last resort.

We present a parameter synthesis algorithm for planar, higher pair systems. We address these systems because they are common in applications, yet receive little attention in prior parameter synthesis research. The input to our algorithm is a parametric model with an initial parameter hyper-box. The algorithm searches the hyper-box for points that cause incorrect function. It excludes these points by revising the hyper-box. Following the robust paradigm, the hyper-box is moved when possible and is shrunk as a last resort.

We cannot guarantee that all incorrect points are found because a complete search of the parameter hyper-box is prohibitively slow. Typical systems have tens to hundreds of parameters. Tiny steps are required because the kinematic function can vary suddenly or even discontinuously. Instead, we search the portion of the parameter hyper-box where problems are most likely. We repeat the search/revision cycle until no incorrect points are found.

Incorrect function is most likely at points that maximize the variation of one or more contacts. Each contact generates a part of the kinematic function. A maximal variation in one contact represents a maximal deviation in its part of the kinematic function. A maximal variation in two or more contacts represents a maximal likelihood for unintended interactions among them. These heuristic arguments are the rationale for our parameter space search algorithm.

The paper is organized as follows. Section 2 reviews prior work. Section 3 specifies the input to our algorithm. Section 4 defines kinematic function for higher pair systems. Section 5 describes our algorithm and Section 6 validates it on five real-world examples. The validation shows that the algorithm finds and corrects incorrect kinematic function with a moderate number of iterations. The paper concludes with a discussion of our results.

2. Prior work

Algorithmic support for parameter synthesis is limited. Commercial packages, such as CATIA and IDEAS, support construction and visualization of parametric designs. Prior research provides synthesis algorithms for linkages [4,5] and cams [6,7], but does not address general higher pair systems. Most parameter synthesis research focuses on tolerance analysis algorithms [8]. The analysis falls into three increasingly general categories: small displacement analysis, large displacement analysis, and contact change analysis.

Small displacement analysis, also referred to as tolerance chain or stack-up analysis, is the most common. It consists of identifying a critical dimensional parameter (a gap, clearance, or play), building a tolerance chain based on part configurations and contacts, and determining the parameter variability range using vectors, tensors, or matrix transforms [9,10]. Recent research explores stack-up analysis with limited contact changes [11–13].

Large displacement analysis has been thoroughly studied in mechanical engineering [4]. It consists of defining kinematic relations between parts with a fixed contact topology, typically a linkage, and studying their kinematic variation [14]. Commercial computer-aided tolerancing systems include this capability for planar and spatial mechanisms [15]. The kinematic variation is computed by linearization, which can be inaccurate, or by Monte Carlo simulation, which can be slow. Glancy and Chase [16] describe a hybrid algorithm that computes the first two derivatives of the system function with respect to the tolerance variables, calculates the first four moments of the system function, and fits an empirical variation distribution.

Large displacement analysis is inappropriate for higher pair systems with many contact changes, such as the examples in this paper. The user must enumerate the contact sequences, analyze them with the software, compose the results, and detect failures. We [17,18] developed the only general kinematic tolerance analysis algorithm for higher pair planar systems.

2.1. Tolerance synthesis

Robust parameter synthesis is related to tolerance synthesis, which is the task of computing optimal tolerance intervals for a given nominal design. Prior work on tolerance synthesis in mechanical systems dates back to 1970, and proposes a variety optimization and modeling criteria [8]. Most work concentrates on modeling the cost-tolerance relations and the formulation of the optimization problem. The modeling limitations of these systems are the same as those for tolerance analysis. In addition, cost and manufacturing processes must be modeled. Dong and Gary [19] survey the issues involved in automating cost tolerance modeling. A general framework, the feasibility space approach, is proposed by Turner [20,21]. However, he assumes that the functional model is given, which is impractical for most mechanical systems. Several researchers develop functional model derivation and optimization algorithms for planar linkages [22,23], and some commercial CAT systems provide this capability [15]. We are unaware of prior work on general tolerance synthesis algorithms for higher pair systems.

2.2. Our prior work

Parameter synthesis is the latest step in our ongoing research on algorithmic mechanical design with configuration spaces. Prior research addresses kinematic analysis [24], kinematic simulation [25], kinematic tolerance analysis [17], and kinematic synthesis [26]. That research provides several analysis tools, described below, that are employed by our parameter synthesis algorithm. However, neither our prior work nor any other published work gives a parameter synthesis algorithm for higher pair systems. The closest prior work is our kinematic synthesis algorithm, which interactively revises a nominal parametric design to eliminate kinematic failures. The designer suggests changes in kinematic function and the program achieves them by parameter space search. The current