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An efficient, error-bounded approximation algorithm for simulating quasi-statics of complex linkages

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Abstract

Design and analysis of articulated mechanical structures, commonly referred to as linkages, is an integral part of any CAD/CAM system. The most common approaches formulate the problem as purely geometric in nature, though dynamics or quasi-statics of linkages should also be considered. Existing optimal algorithms that compute forward dynamics or quasi-statics of linkages have a linear runtime dependence on the number of joints in the linkage. When forces are applied to a linkage, these techniques need to compute the accelerations of all the joints and can become impractical for rapid prototyping of highly complex linkages with a large number of joints.

We introduce a novel algorithm that enables adaptive refinement of the forward quasi-statics simulation of complex linkages. This algorithm can cull away joints whose contribution to the overall linkage motion is below a given user-defined threshold, thus limiting the computation of the joint accelerations and forces to those that contribute most to the overall motion. It also allows a natural trade-off between the precision of the resulting simulation and the time required to compute it. We have implemented our algorithm and tested its performance on complex benchmarks consisting of up to 50,000 joints. We demonstrate that in some cases our algorithm is able to achieve up to two orders of magnitude of performance improvement, while providing a high precision, error-bounded approximation of the quasi-statics of the simulated linkage.

Keywords: Design and analysis; Simulation; Virtual prototyping

1. Introduction

Modeling and design of articulated mechanical structures, commonly referred to as 'linkages', is an integral part of any CAD/CAM system. The existing approaches typically focus on sizing mechanical constraints that guide the movement of the end-effector of the system within a workspace. The function of the device is usually prescribed as a set of positions reachable by the end-effector, as well as the mechanical constraints formed by joints that limit relative movement. The objective is to compute all the linkage configurations that can achieve a specific task. Formulated in this way the design problem is purely geometric in nature, often with an emphasis on systems consisting of lower numbers of degrees of freedom to reduce the complexity of the problem.

Complex linkages, such as articulated systems of many links, molecular chains, or articulated-body representations of deformable bodies, often require consideration of additional physical constraints and present new computational challenges. Forward simulation of articulated mechanical systems, i.e. computing the joint accelerations and forces depending on external forces and applied torques, is a fundamental issue that should be taken into account in designing highly complex linkages. Especially, a quasi-static analysis can provide insights into mechanisms at or close to equilibrium, or which have small dynamic effects (slow evolution or small-scale motions). However, such computations are frequently expensive to perform and sometimes ignored in rapid prototyping of highly complex mechanical linkages. Existing optimal algorithms that compute forward dynamics or quasi-statics of linkages have a linear runtime dependence on the number of joints in the linkage. When forces are applied to the linkage, these techniques need to compute the accelerations of all the joints, which can become impractical for rapid prototyping of complex linkages with a large number of joints.

In this paper, we present a novel algorithm for progressive refinement of the quasi-static simulation of complex linkages, which offers three key advantages:

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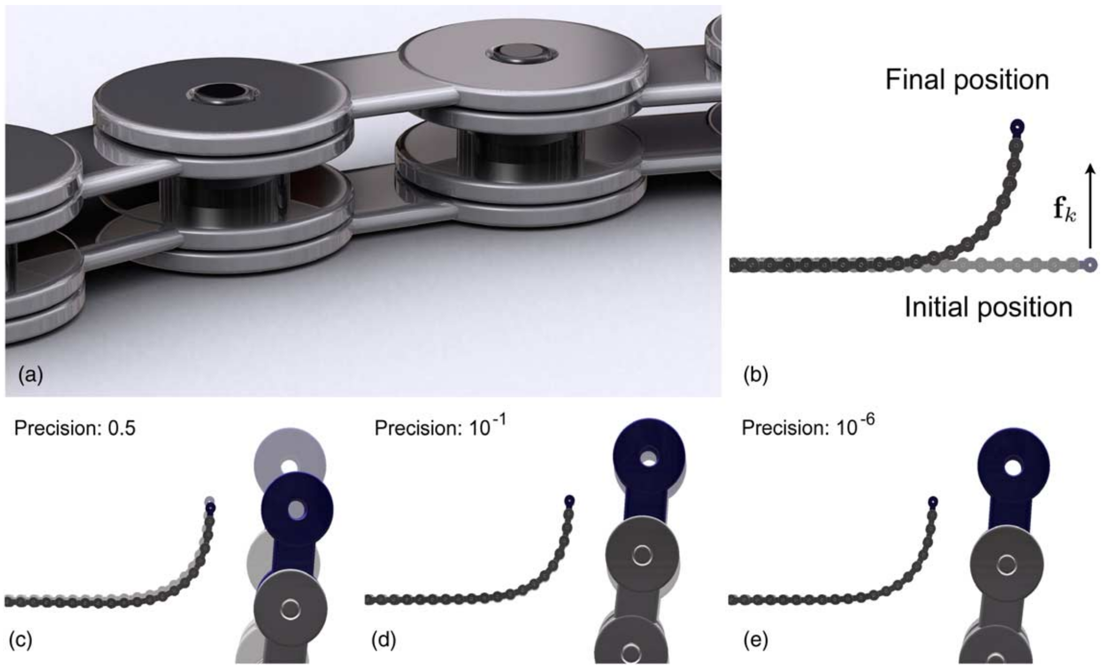

Fig. 1. Error-bounded approximation of a complex linkage's quasi-statics. (a) A close-up view of the model. (b) The initial and final positions of the linkage. (c)–(e) The simulation results of our algorithm vs. the DCA for varying degrees of precision, with both the view of the chain and a close-up of the end position.

- *Error-bounded approximation*. Given any user-defined error threshold, our algorithm is able to determine an approximation of the linkage acceleration within the required error bound.
- *Progressive refinement*. Our algorithm operates by progressively increasing the quality of the approximate linkage acceleration, thus providing a trade-off between the precision of the approximation and the time required to compute it.
- *Efficiency*. Our algorithm determines the approximate acceleration by computing only some of the joints' accelerations and associated coefficients, resulting in a significant performance improvement when the ratio on the number of processed joints to the total number of joints is small, i.e. when the required precision is low, or when the forces applied to the linkage have a local effect only on the linkage motion.

To the best of our knowledge, this algorithm is the first technique that offers a method to progressively refine the computation of the quasi-statics of a linkage while guaranteeing the precision of the calculations. We have implemented our algorithm in C++ and tested it on some complex mechanical systems consisting of up to 50,000 links. In some cases, we were able to achieve up to two orders of magnitude of performance improvement over an existing linear-time algorithm, while providing a high precision, error-bounded approximation of the quasi-statics of the simulated linkage (see an example in Fig. 1).

The rest of the paper is organized as follows. Section 2 presents a brief survey of related work on rigid body systems. Section 3 gives an overview of the basic mathematical notation and representations that are fundamental to our approach. Section 4 demonstrates how we obtain an approximate acceleration of a linkage. Section 5 shows how, using the hierarchical reference frames representation we introduce, the linkage state update can be restricted to some coefficients only. Section 6 presents some results obtained with our new approach, and Section 7 concludes by suggesting some potential research directions.

2. Related work

The computation of the kinematics, quasi-statics, or dynamics of articulated bodies has been a subject of study for many years (cf. [1] for example). We only provide a brief overview here and refer the readers to [2] for a recent state-of-the-art survey.

Essentially, an important step in the development of simulation algorithms has been the introduction of methods, which have a linear complexity in the number of bodies in the multibody system. Some of these linear methods [3–7], which rely on a recursive formulation of the motion equations, have now been known for more than two decades. In parallel, several authors have contributed to simplify the motion equations by developing new notations and formulations. Some of these include the spatial notation [5], and its new version [8,9], the spatial operator algebra [10], as well as Lie-Group formulations [11]. More recently, several researchers have proposed parallel algorithms to compute the forward dynamics of articulated bodies using multiple processors [8,9,12].

To our knowledge, no algorithm has been proposed for obtaining an error-bounded approximation of the quasi-statics of a multibody system. All existing techniques compute the