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# Arc–surface intersection method to calculate cutter–workpiece engagements for generic cutter in five-axis milling

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ABSTRACT

Calculating cutter–workpiece engagements (CWEs) is essential to the physical simulation of milling process that starts with the prediction of cutting forces. As for five-axis milling of free form surfaces, the calculation of CWEs remains a challenge due to the complicated and varying engagement geometries that occur between the cutter and the in-process workpiece. In this paper, a new arc–surface intersection method (ASIM) is proposed to obtain CWEs for generic cutter in five-axis milling. The cutter rotary surface is first represented by the family of section circles which are generated by slicing the cutter with planes perpendicular to the tool axis. Based on the envelope condition, two grazing points on each section circle are analytically derived, which divide the circle into two arcs. The feasible contact arc (FCA) is then extracted to intersect with workpiece surfaces. Using arc/surface intersection and distance fields based approach, the boundary of the closed CWEs is accurately and efficiently calculated. Compared with the solid modeler based method and the discrete method, the ASIM has higher computational efficiency and accuracy. Moreover, an analytical solution for calculating CWEs can be obtained with this method in five-axis milling of the workpiece merely comprising of flat and quadric surfaces. Finally, two case tests are implemented to confirm the validity of the ASIM and comparisons have been made with a Vericut based system which utilizes the Z-buffer method. The results indicate that the ASIM is computationally efficient, accurate and robust.

## 1. Introduction

Five-axis milling has been widely used in the production of complex parts found in aerospace, automotive, die-mold and biomedical industries [1–5]. The machining process is usually costly and time-consuming especially for high accuracy required parts. Hence, the main focus of five-axis machining is to reduce cycle times while ensuring manufacturing precision. Generally, optimal cutting parameters are selected by the industry through simulating the physics of the cutting process, i.e. cutting force and chatter stability. As for the machining process simulation and optimization, one of the most important steps is to extract the cutter–workpiece engagements (CWEs) which are the portions of the cutter participating in machining at a given instant of time.

Due to the complex geometry of the workpiece and the varying tool motions, the accurate and efficient way to calculate CWEs of five-axis milling has been very limited. Some researchers tried to extract the CWEs in analytical ways. Gupta et al. [6] presented an analytical method to determine the CWE functions with the half-spaces in the following cases: circular cut and linear half-space. Budak et al. [7] proposed the bounding point coordinate method to calculate the depth of cut, lead, and tilt angles, which determine the CWE boundaries in five-axis milling. Kiswanto et al. [8] calculated the CWEs during the semi-finish milling process by finding the lower engagement point and the upper engagement point. However, these methods are only applicable for milling of flat workpiece surfaces. Since the workpiece in five-axis milling usually consists of complex shapes, the usage of the analytical approaches is limited.

For the calculation of CWEs in five-axis milling, the most common approaches can be classified into two major categories: discrete modeling methods and solid modeling methods. Discrete modeling approaches can further be categorized into two groups: vector based methods and polygon based methods. Vector modeling divides the workpiece into finite units with regular interval methods well known as Z-map [9], Z-buffer [10,11], Dexel [12] and Octree [13]. The commonly used Z-map method represents the workpiece as a series of evenly distributed parallel vectors. The length of these vectors is reduced while the cutter

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moves through the workpiece which is similar to cutting blades of grass [14]. Based on this method, several studies have been conducted to verify the correctness of tool paths and support the physical simulation for both three-axis and five-axis milling. Baek et al. [15,16] used the Z-map method to perform tool path verification through calculating the intersection points between vectors and the tool swept surface in three-axis milling. Zhu et al. [17], Fussell et al. [10] and Zhang [18] model the CWE geometry to predict cutting force for five-axis sculptured surface milling using vector based method. Polygon based methods have also received some attention [19,20]. Aras et al. [19] presented a methodology that maps a polyhedral representation of the removal volume from a Euclidean space into a parametric space to find CWEs for three-axis milling. Yao et al. [20] developed a hybrid approach that utilizes an exact model of a cutter to intersect with a tessellated model of a workpiece. Though computationally efficient and mathematically tractable, these techniques suffer from the contradiction between the dispersed precision of workpiece decomposition and the computational efficiency. In other words, the high accuracy computation of CWEs requires the high discrete resolution of the workpiece which comes with the expense of large store memory and computational requirements.

Solid modeling offers a high level of accuracy for NC simulation. It is now widely used in CAD and CAM areas with the developing computer technology. The commonly used solid modeling representation schemes are constructive solid geometry (CSG) and boundary representation (B-Rep). Spence et al. [21,22] identified CWEs using a CSG based process simulation system to predict cutting force. Recently, Lazoglu et al. [23,24] proposed a novel B-Rep based method to determine the complex CWEs in supporting five-axis milling of free-form surfaces. Aras et al. [25] obtained the closed boundaries of the CWEs by performing surface/surface intersections between in-process workpiece and feasible contact surfaces (FCS). Although solid modeling methods have been recognized as the most accurate approach to extract CWE geometry, it is far from being widely used in practical application due to its low computational efficiency and poor robustness. For instance, the computational complexity of the CSG approach is $O(N^4)$, where $N$ denotes the number of tool movements [26]. What is worse, as the data structure size grows quickly during simulation, topological errors due to numerical inaccuracy will be stacked [27]. To overcome such problems, Ferry et al. [28] proposed a semi-discrete solid modeling technique called parallel slicing method (PSM) for five-axis flank milling where the removal volume is sliced into a number of parallel planes along a common axis. Yang et al. [27] developed a solid trimming method which reduces the abundant surface/surface intersection operations compared with the traditional Boolean operations. However, the improvements of these approaches in computational efficiency and robustness are limited.

In this paper, we present a new arc–surface intersection method (ASIM) to extract CWEs in five-axis milling process. First, the cutter rotary surface is sliced into a family of section circles by a set of planes perpendicular to the tool axis. Then, based on the envelope condition, the grazing points of the family of section circles are analytically computed. By connecting the two grazing points on each circle, a feasible contact arc (FCA) which locates at the front along the feed direction is obtained. Engagement arcs are extracted accurately and efficiently by intersecting the FCA with workpiece surfaces and determining whether the endpoint of the FCA is inside the workpiece volume based on distance fields. Finally, entry/exit angles of CWEs at each axis height are obtained to plot the CWE map which can be directly used for cutting force prediction. As the boundary of CWEs at each axis height is accurately determined by the arc/surface intersection rather than the abundant Boolean operation, this method well solves the problems of poor computational efficiency and robustness in solid modeling methods and low computational accuracy in discrete methods. Comparisons between our methodology and those commonly used in the literature are presented in Table 1.

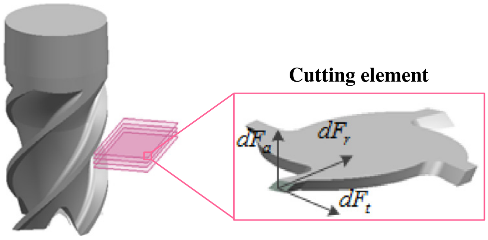

**Fig. 1.** Discretization of the cutter into a set of axial cutting elements.

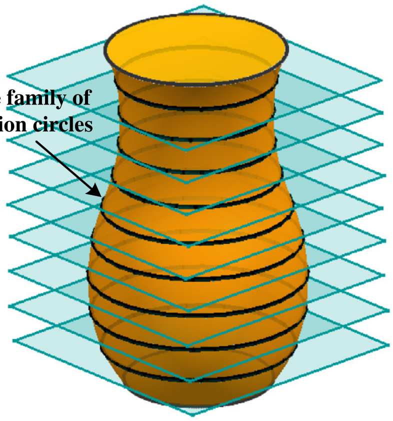

**Fig. 2.** Axial discretization of the generic cutter surface.

The remainder of this paper is organized as follows: Generation of the feasible contact arcs is described in Section 2, followed by the five-axis CWE extraction methodology in Section 3. Implementations are given in Section 4, and conclusion is summarized in Section 5.

## 2. Generation of the feasible contact arcs

For the convenience of study, the commonly used cutting force model in five-axis milling always divides the cutter into a set of axial cutting elements [17,21,29], as illustrated in Fig. 1. To judge whether or not the cutting element is in cut at any angular position, CWE regions should be first extracted and then mapped onto a parametric space defined by the entry/exit angles versus the depth of cut. To avoid the time consuming surface/surface intersection operation in the solid modeling method, in this paper, the cutter rotary surface is discretized into a family of section circles which are generated by slicing the cutter with planes perpendicular to the tool axis, as shown in Fig. 2. Then, the section circle is further used as the primitive to extract the CWEs.

In five-axis milling, the grazing curve is the set of points on the cutter surface that remain on the envelope surface. As shown in Fig. 3, it divides the cutter's surface into two parts. The one in orange facing the cutting direction is defined as the front-facing part, which can be actually involved in machining. The other one in light gray is defined as the back-facing part, which is impossible to engage with the in-process workpiece.

For the section circle on the cutter surface, it is splitted into two partial arcs by the grazing points. The solid one shown in Fig. 3 is named as the feasible contact arc (FCA) since it locates at the tool front-facing part. Given that the CWEs are the subsets of the tool front-facing part rather than the back-facing part, determination of the FCA helps us to obtain CWEs in a more efficient way.